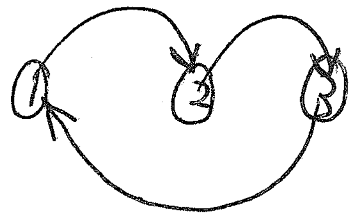
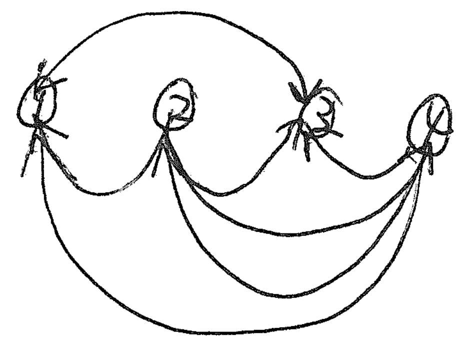
Chapter 5

3. oriented graph



4. A_1 has oriented graph



no connection from 1 to 2 or 4

not strongly connected, A_1 reducible

A_2 has oriented graph which could be represented as permutation

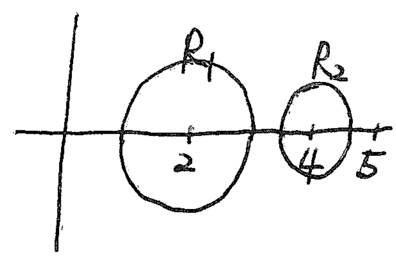
$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$ it has order 4, so the oriented graph is strongly connected.

5. Do permutation $(2,3)(3,4)(4,5)(1,2)A(1,2)(4,5)(3,4)(2,3)$

A can be reduced to

$$\left(\begin{array}{cc|ccc} 2 & 1 & 2 & 4 & -3 \\ 0.5 & 4 & 2 & -1 & 3 \\ \hline 0 & 0 & -4 & 0 & 0.5 \\ 0 & 0 & 0.5 & -1 & 0 \\ 0 & 0 & 0.5 & 0.2 & 3 \end{array}\right)$$

$M_1 = \begin{pmatrix} 2 & 1 \\ 0.5 & 4 \end{pmatrix}$

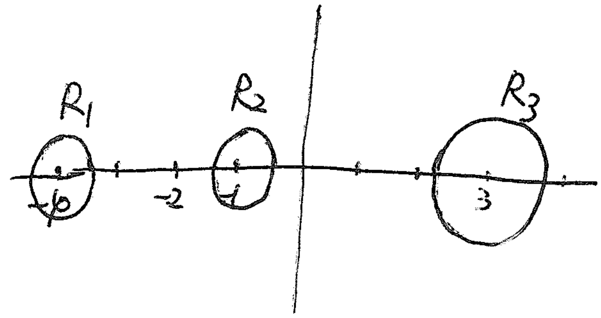


R_1 R_2 are separated, thus each contains one eigenvalue.

Since A is real. the complex eigenvalues must appear as conjugate pair.

so the eigenvalues in R_1 and R_2 must be real.

$$M_2 = \begin{pmatrix} -4 & 0 & 0.5 \\ 0.5 & -1 & 0 \\ 0.5 & 0.2 & 3 \end{pmatrix}$$



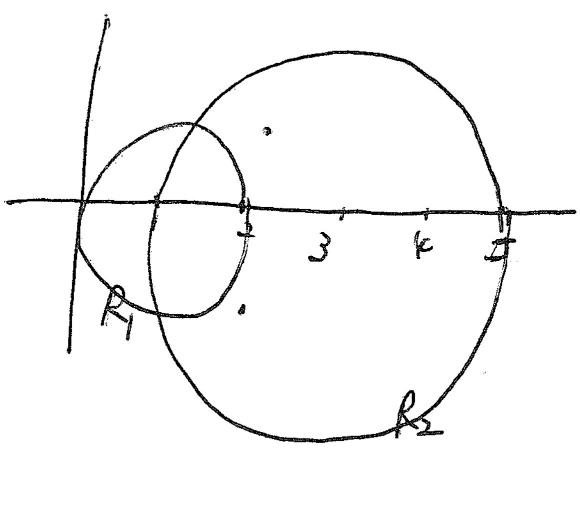
R_1 R_2 R_3 are separated thus each contains one eigenvalue

and they must be real.

6. Wrong Problem

Example: $\hat{A} = \begin{pmatrix} 1 & 1 \\ -2 & 3 \end{pmatrix}$ has eigenvalues $2 \pm i$

they are both outside R_1



Gershgorin Disk theorem could not guarantee each disk contains eigenvalue. unless they are separated.

10. $X = \sum_{i=2}^{n} c_i X_i$

$$A^k X = \sum_{i=2}^{n} c_i \lambda_i^k X_i = \lambda_2^k \left(c_2 X_2 + c_3 \left(\frac{\lambda_3}{\lambda_2}\right)^k X_3 + \cdots c_n \left(\frac{\lambda_n}{\lambda_2}\right)^k X_n \right)$$

Since $|\lambda_1| > |\lambda_2| \cdots > |\lambda_n|$

$c_3 \left(\frac{\lambda_3}{\lambda_2}\right)^k X_3 + \cdots c_n \left(\frac{\lambda_n}{\lambda_2}\right)^k X_n \to 0$ thus $q_k \to X_2$