Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
TOTAL:	

Name and TA Section:	
Student Number:	

SAMPLE Midterm #1, Math 20D, 2004 Fall Quarter

Place/Time: PCYNH/MULTI 106, 9:00-9:50am, 22 October 2004

Instructions: Solve the following five problems. If you need extra paper, first use the back of each page of the exam. If necessary, additional paper can be provided and should be stapled to the exam (in this case, clearly indicate which problem you are solving). (NOTE: Anything in red would not appear on a real exam.)

Problem 1. (20 points – 5 points for each part)

(a) [11.1: Sequences] Give the mathematical definition of what is meant by: $\lim_{n\to\infty} a_n = L$.

(b) [11.2: Series] Give the mathematical definition of what is meant by: $\sum_{n=1}^{\infty} a_n = S$.

(c) [11.2: Series] If it is known that $\sum_{n=1}^{\infty} a_n = S$, what can be said about $\lim_{n\to\infty} a_n$?

(d) [11.10: Taylor and Maclaurin Series] Given an analytic function $f(x)$, write down its general Taylor series centered at a.

Problem 2. (20 points) [11.1: Sequences] Determine whether the following *sequence* converges or diverges. If it converges, find the limit.

$$a_n = 2 + \frac{(-1)^{n+1} 8^{n+2}}{9^n}, \qquad n = 1, 2, 3, \ldots$$

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Problem 3. (20 points) [11.4-11.6: Comparision Test, Alternating Series, Absolute Convergence] Is the following *series* absolutely convergent, conditionally convergent, or divergent? (Justify your answer.) If it converges, estimate $R_3 = S - S_3$, which is the remainder error when using the 3rd partial sum S_3 as an approximation to the sum of the series S.

$$\sum_{n=1}^{\infty} \frac{2(-1)^n}{3+n}$$

Problem 4. (20 points) [11.8,11.10: Power Series, Taylor and Maclaurin Series] Starting with the general form of a Taylor series, *derive* the MacLaurin series for $f(x) = 2e^{3x}$, and show that it converges for all x. Furthermore, show that $f(x)$ is actually *analytic*, by showing it is actually the sum of its MacLaurin series. (Hint: To show that $f(x)$ is analytic, use Taylor's inequality and the Squeeze Theorem to show $\lim_{n\to\infty} R_n(x) = 0$, where $f(x) = T_n(x) + R_n(x)$, with $T_n(x)$ the n-th degree Taylor polynomial and $R_n(x)$ the remainder.)

Problem 5. (20 points) [1.1-1.2: Simple Linear ODE Models of Growth and Decay] Find the general solution of the following ODE, then solve the associated IVP. (Hint: After finding the general solution, use the initial condition to determine the integration constant appearing in the general solution.)

$$2y' + 4ty = 0, \qquad y(0) = 3.$$

Finally, evaluate the solution at $t = 3$.