Discussion 3: July 26th, 2017

Instructions: The class is split into groups of 3 students; the TA will have a list of the groups. As a group, you will answer several questions, one of which will be turned in for credit. You will each have an assigned role in your group. The roles are:

1. Manager - Keep your group “on-track.” Make sure everyone participates. Watch the time spent on each problem.
2. Skeptic - Help your group avoid coming to agreement too quickly. Make sure all possibilities are explored. Suggest alternative ideas.
3. Recorder/checker - Act as a scribe for your group. Check for understanding from all group members. Make sure all group members agree. Turn in the assignment to Gradescope and make sure all group member names are included in Gradescope!
4. (If a 4th student) Energizer/summarizer - Energize your group when motivation is low by suggesting a new idea, using humor or being enthusiastic. Summarize your group’s discussion and conclusions.

The first question set you will not turn in. The second question set you will turn in. Those question(s) will be graded half as participation points. Solutions for those question(s) will be posted on the course website.

As a group, complete the question(s) that will not be turned in first. Your TA will confirm that you have the correct answer, and will initial your paper for the solutions you will turn in. Once you have that, work on the question that will be turned in. The recorder will write the group’s answer on the initialed paper (and more paper, if you need), then will take one (or more) photo(s) of it to turn it into Gradescope. Submitting this is like submitting homework, but you will be able to add additional students to your submission, after you upload the photo. Make sure all students’ names are on the solution sheet! Also make sure to add all students in your group in Gradescope so they get credit! Then, spend a few minutes as a group discussing how your group work went: what went well and what you each could do better next time. (Be polite.) Once you are done, you may continue to work on homework questions, or you may leave.

Not to be turned in:

1. Set up and solve the following least squares problem. (You can just use the normal equations, which are easier to do by hand than the QR decomposition method.) The data is \((x, y) = (3, 2), (2, 5), (5, 8)\) and your model is \(y = a_1 + a_2(x - 3)^2\). (I did not choose the numbers to come out nicely.)

2. Calculate the QR decomposition of \(A = \begin{bmatrix} 5 & -2 \\ 12 & 3 \\ 0 & 4 \end{bmatrix}\). (Your \(R\) should end up with integer entries.)

Discussion 3: Questions to be turned in.

Make sure to write your names on the sheet(s) with your solutions! Also, get your TA to initial it to show you did the earlier problems.

1. Explain why the solution of the normal equation \(A^T A x = A^T b\) is the solution to the least squares problem \(A x = b\). You may assume that theorem 3.5.15 (and its proof) are already understood. That theorem says that \(\|b - y\|_2\) is minimized (over any choice of \(y \in S\)) when \(y\) is the projection of \(b\) onto \(S\). In other words,

\[
\|b - y\|_2 = \min_{s \in S} \|b - s\|_2.
\]

You can also assume that \(R(A)^\perp = N(A^T)\).

2. When calculating the QR decomposition with reflections, it is inefficient to store and use the matrices \(Q_i\). Explain what the efficient way to store and use \(Q\) is, and why the naive way of storing the \(Q_i\) is inefficient.