Lecture 12 (2.3, 2.4)

Math 170A

October 30, 2017
Find a matrix $\delta A$ so that $A + \delta A$ is singular for $A = \begin{bmatrix} 1 & 1 \\ 1 & 1.01 \end{bmatrix}$.
Try to make $\delta A$ small.
Suppose $A = \begin{bmatrix} 1 & 1 \\ 1 & 1.01 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 101 & -100 \\ -100 & 100 \end{bmatrix}$. If $\|\delta A\|_1 \leq 0.001$, about how large could our error be from solving $(A + \delta A)(x + \delta x) = b$? (Use the 1-norm.)

(a) Somewhere around 0.2% error.
(b) Somewhere around 2% error.
(c) Somewhere around 20% error.
(d) Somewhere around 200% error.
(e) None of these are correct.
Suppose $A = \begin{bmatrix} 1 & 1 \\ 1 & 1.01 \end{bmatrix}$, and so $\kappa_1(A) \approx 400$. If I solve $Ax = b$ for $b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, and my residual is $\hat{r} = \begin{bmatrix} 2 \cdot 10^{-5} \\ -1 \cdot 10^{-5} \end{bmatrix}$, how inaccurate could my solution be? (Use the 1-norm.)

(a) Somewhere around 0.4% error.
(b) Somewhere around 4% error.
(c) Somewhere around 40% error.
(d) Somewhere around 400% error.
(e) None of these are correct.