Homework 8: Due November 27th, 2017

First, do these problems. These will be graded for completion.

- 4.1: 6, Math majors should do 16, but don’t turn it in.
- 4.2: 5, 14
- 5.2: 2, 14a,b, 17
- 5.3: 6, 8, 13, 14, 15, 45

Then do these problems. These will be graded for correctness.

1. Watkins 4.2.10. ALSO: Use the results of this question to explain why one should use the QR decomposition to solve the least squares problem instead of the normal equations.
2. Explain why iterative methods are necessary to find eigenvalues and eigenvectors of matrices.
3. Write a function in MATLAB that takes as input a square matrix \( A \), a guess for an eigenvalue \( \rho \) and returns as output the eigenvalue and eigenvector found using the shift-and-invert strategy. You may use MATLAB’s built in matrix multiplication (\( A \cdot b \)) and “division” command (\( A \div b \)) as necessary. To test your program, if you enter the matrix \( A = \begin{bmatrix} 1 & 2 \\ 2 & 8 \end{bmatrix} \) and \( \rho = 1 \), you should find the eigenvalue 0.19825 and the eigenvector \((-0.33455, 1, -0.57726)\).
4. Explain why the basic power method \( q, Aq, A^2q, \cdots \) (with scaling) converges to the eigenvector associated with the largest eigenvalue. (You can assume that \( A \) is semisimple.)
5. Let \( A = \begin{bmatrix} 1 & 2 \\ 2 & 8 \end{bmatrix} \). This matrix’s dominant eigenvalue is approximately 8. Start from the vector (0, 1) and use the power method (by hand, for 3 iterations) to estimate the dominant eigenvalue and its associated eigenvector.

For additional practice, here are some optional problems. These should not be turned in.

- 4.1: 5
- 4.2: 17, 20
- 5.2: 6, 7, 13, 18
- 5.3: 7, 10, 17, h