Homework 10: Due December 11th, 2017

First, do these problems. These will be graded for completion.

- 8.3: 10, 12, 17a, 19a,b, 20, 22
- 8.4: 7a, 12
- Usually it doesn’t matter which stopping criterion you use. Explain briefly why sometimes it does.
- Briefly and intuitively explain why preconditioners help descent methods converge more quickly.

Then do these problems. These will be graded for correctness.

1. Watkins 8.4.8(a,b). Use a sketch like Figure 8.4 to explain why part b makes sense graphically, and why overrelaxation performs better than standard Gauss-Seidel, but only for a good choice of $\omega$. (This picture, to be clear, only applies when $A$ is positive definite.)

2. Write a function in MATLAB that takes as input a symmetric positive definite matrix $A$, the right hand side vector $b$, a maximum number of iterations, and a tolerance for convergence and returns as output the solution of $Ax = b$ as found by performing the conjugate-gradient method as found in (8.7.1). It will be easiest to program the exact algorithm found there, though you will have to change notation and names of variables so that MATLAB can understand it, and check the tolerance.

3. Compare your SOR solver from the last homework assignment with your conjugate-gradient method. Pick a large matrix (such as the one for the Poisson equation), and compare the number of iterations each method uses to solve $Ax = b$ up to the desired precision. (Compare with several $\omega$.)

For additional practice, here are some optional problems. These should not be turned in.

- 8.4: 21
- 8.7: 9