Problem 0.1. Watkins 2.1.28(a)

Solution. (a) \( \|I\|_F = \sqrt{\sum_{i=1}^{n} i} = \sqrt{n} \), and \( \|I\|_2 = \max \frac{\|Ax\|}{\|x\|} = \max \frac{\|x\|}{\|x\|} = 1. \)

Problem 0.2. Calculate \( \|A\|_{\infty} \) and \( \|A\|_1 \) if \( A = \begin{bmatrix} 1 & -1 \\ 3 & 4 \end{bmatrix} \).

Solution. \( \|A\|_{\infty} \) is the biggest row, and so is 7. \( \|A\|_1 \) is the biggest column, and so is 5.

Problem 0.3. What intuitive quantity does a vector norm measure? Using that interpretation, explain why the three required properties for a norm are properties that should “obviously” hold for that quantity.

Solution. A vector norm intuitively measure the length or distance of a vector.

The first property is that \( \|\vec{x}\| > 0 \) unless \( \vec{x} = \vec{0} \). This is obvious since, unless your thing is zero, it should have positive length. (It shouldn’t have zero or negative length, since that doesn’t make sense.)

The second property is that \( \|\alpha\vec{x}\| = |\alpha|\|\vec{x}\| \). This is obvious since if it’s twice as big, the length should be twice as long.

The last property is that \( \|\vec{x} + \vec{y}\| \leq \|\vec{x}\| + \|\vec{y}\| \). This is obvious because it says that the shortest path is the straight one, if we think of these vectors as being standard vectors in the plane.