Day 2: 5.1 - Read instructions. Please

As I said last time, main goal last term was function ⇒ rate of change or deriv.

This term: rate of change or deriv ⇒ function

The standard example last term was position and speed/velocity.

derv/rate of change of position is speed/velocity.

So, the standard example the other way is Velocity ⇒ position.

Q1: If you travel 100 Km/hr for 15 min, how far do you travel?

\[
\frac{1}{4} \text{ hr} \cdot 100 \frac{\text{km}}{\text{hr}} = 25 \text{ km} \text{. easy.}
\]

If the speed is constant, figuring out distance is easy. distance = velocity × times.

What if it's changing? (You don't drive the same speed for an entire trip!)

Well, you could look at your odometer, of course.

But also, could break your overall trip into small segments:

<table>
<thead>
<tr>
<th>time (hr)</th>
<th>0</th>
<th>1/2</th>
<th>1</th>
<th>3/2</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>velocity</td>
<td>100</td>
<td>60</td>
<td>80</td>
<td>100</td>
<td>120</td>
</tr>
</tbody>
</table>

(start fast, hit some traffic, try to make up for it)

Q2: About how far did you travel in 2 hours? (include double correct)

For first half hour, probably travelling b/w 100 and 60 Km/hr, but let's say 100 Km/hr.

So traveled \(100 \frac{\text{Km}}{\text{hr}} \cdot \frac{1}{2} \text{ hr} = 50 \text{ Km.}\)

For 2nd half hour, probably b/w 60 and 80 Km/hr, but say 60 Km/hr

\(60 \frac{\text{Km}}{\text{hr}} \cdot \frac{1}{2} \text{ hr} = 30 \text{ Km}\)

Similarly get 40 Km and 50 Km, (left estimate) book says "lower" estimate but

Add up, get \(\approx 170 \text{ Km traveled.} \) (left estimate) But only under special conditions, I don't care usually, but just pick one.

OR could use 60 km/hr for 1st half hour, 80 for 2nd, etc,

get \(\approx 30 + 40 + 50 + 60 = 180 \text{ Km traveled.} \) (right estimate)

Two possible estimates. Probably between.

Of course, if I had more times when we measured speed, we could estimate better.

But let's graph what we just did:

\[
\text{distance} = 100 \frac{\text{Km}}{\text{hr}} \cdot \frac{1}{2} \text{ hr} \approx \text{like rectangle}
\]

In both cases, we approximated the distance by approximating the area under the curve.
Q3: Explain why "area" gives "distance traveled."

This is key idea: These rectangles have "height" in Km/hr, "width" in hr.
So "area" is "height" x "width" = \( \frac{Km}{hr} \times hr = Km. \)
And that multiplication is how you estimate distance traveled.

The book spends more time than is important for this class worrying about the exact size of the error.
What is important is that smaller intervals of time (usually) lead to smaller errors.

But, area under curve is total distance.

Key idea: The area under a velocity curve gives total change in position.

Why do I say "total change" rather than "distance traveled"? Because sometimes we go backwards:

Q4: Going to campus, realize you forgot your clicker. Run back to apartment. Neg represents back.

\[
\begin{array}{c|c|c|c|c}
\text{time} t & 0 & 1 & 2 & 3 \\
\text{km/min} v(t) & 0 & 1 & -2 & 0 \\
\end{array}
\]

How far did you travel? \( \approx \frac{2}{10} \times \frac{9}{5} = \frac{2}{5} \text{ km} \)

What was your total change in position? \( \approx 0 \).

The area under curve represents distance traveled in other direction.

A bit of notation:

\[
\begin{array}{c|c|c|c|c}
\text{time} t & t_0 = 0 & t_1 = 1 & t_2 = 2 & t_3 = 3 \\
v(t) & V(t_0) = 0 & V(t_1) = \frac{1}{10} & V(t_2) = -\frac{2}{5} & V(t_3) = 0 \\
\end{array}
\]

\[
V(t_0) \cdot \Delta t
\]
Approximation is speed x time

Speed at t. change in time

So "left" estimate is \( V(t_0) \Delta t + V(t_1) \Delta t + V(t_2) \Delta t \)

"Right" estimate is \( V(t_1) \Delta t + V(t_2) \Delta t + V(t_3) \Delta t \)
**Key idea**: \( \int_a^b f(x) \, dx \) is the "signed" area under the graph of \( f(x) \).

- "integral of \( f(x) \) from \( a \) to \( b \)," or "integral from \( a \) to \( b \) of \( f(x) \, dx \)."

- \( f(x) \) is called the integrand.
- \( a, b \) are called the limits of integration.
- What do I mean by "signed" area?

Now, this is not quite the definition. We'll get there.

Q1: \( \int_a^b f(x) \, dx \) is always positive, since area. (False! Already talked about neg. area.)

Q2: Calculate \( \int_1^3 f(x) \, dx \) for this func.

\[
\begin{array}{c|c}
 f(x) & -2 \\
\end{array}
\]

Q3: Calculate \( \int_0^{2\pi} \sin(x) \, dx \). (Hint: sketch the graph.)

\[
\int_0^{2\pi} \sin(x) \, dx = 0
\]

Now back to def'n: "Area under curve" needs to be defined, to be careful.

Idea: integral is the limit of the rectangular approximations of area.

\[
\text{How to write this? areas } = f(t_0) \Delta t + f(t_1) \Delta t + f(t_2) \Delta t
\]

(new notation) \[ \sum_{i=0}^{n} f(t_i) \Delta t \]

Sum of \( \text{from } i = 0 \text{ to } i = 2 \), starting \( i \) \( \to \) last \( i \).
So: "left" approximation with \( n \) rectangles is \( \sum_{i=0}^{n-1} f(x_i) \Delta x \), called "Riemann Sum".

Just the same as before, just new way of writing it.

Q4: Test on notation: What is \( \sum_{i=0}^{3} i \) ? (0, 1, 2, 3, 6, 0, 3.)

\[ \sum_{i=0}^{3} i = 0 + 1 + 2 + 3 = 6 \]

(put 0 in, then put 1 in, etc, then add together.)

Defn of integral: \( \int_a^b f(x) \, dx = \lim_{n \to \infty} \left( \text{"left" approximation} = \lim_{n \to \infty} \left( \sum_{i=0}^{n-1} f(x_i) \Delta x \right) \right) \)

Recall: defn of deriv was limit of approximate (average) slopes.

Defn of integral is limit of approximate areas. (Riemann Sums)

Why this notation \( \int \) is a S! (for sum, just like \( \Sigma \))

\[ \int_a^b f(x) \, dx \]

[add up areas.]

\[ \left( \begin{array}{c} \text{area} \\ \hline \end{array} \right) \]

\[ \text{height} \quad \text{width} \]

(infinite number of)

Now, as before, sometimes we can look at the graph and calculate areas. Usually can't.

Next time, we'll say how to calculate, at least in theory. Though often impossible in practice.

But always can approximate with Riemann Sums!

Q5: Use 4 rectangles to approximate \( \int_0^2 x^2 \, dx \). (a 4-term Riemann Sum?)

\[
\begin{array}{c|ccccc}
\hline
x & 0 & \frac{1}{4} & 1 & \frac{3}{4} & 2 \\
\hline
f(x) & 0 & \frac{1}{4} & 1 & \frac{9}{4} & 4 \\
\hline
\end{array}
\]

\[
\int_0^2 x^2 \, dx \approx 0 \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} + \frac{9}{4} \cdot \frac{1}{2} \\
= \frac{1}{8} + \frac{1}{4} + \frac{9}{8} = \frac{14}{8} = \frac{7}{4}
\]

(true answer is 2, so not bad)

RH approx: \( \frac{1}{4} \cdot \frac{1}{2} + \ldots + 4 \cdot \frac{1}{2} = \frac{15}{4} \)
Units for derivatives: \( f'(x) = \frac{df}{dx} \Rightarrow \) units are \( \frac{\text{units for } f}{\text{units for } x} \).

Notation is suggestive

Units for integrals: \( \int_a^b f(x) \, dx \Rightarrow \) units are units for \( f \), units for \( x \).

Exactly since adding up "areas" of rectangles.

Q1: Acceleration is often measured in \( \text{m/s}^2 \). What are units for \( \int_0^{100} a(t) \, dt \)?

- Units for \( a \) = units for \( \text{m/s}^2 \)
- Units for \( t = \frac{m}{s^2} \cdot \text{s} = \frac{m}{s} = \text{velocity} \)

Similarly, \( \int_0^{100} v(t) \, dt \) has units \( \frac{m}{s} \cdot \text{s} = m = \text{distance} \)

\( \Rightarrow \) back to this example: \( \int_0^{100} v(t) \, dt = \) total change in position from \( t=0 \) to \( t=100 \)

This is heart of Fund. Thm. of Calc.

\[ \boxed{\text{Fundamental Theorem of Calculus:}} \]

\[ \int_a^b F'(x) \, dx = \text{total change in } F(x) \text{ from } x=a \text{ to } x=b = F(b) - F(a) \]

\( \star \) The integral of a rate of change gives the total change. \( \star \)

ex: integral of velocity gives total change of position.

ex: integral of acceleration gives total change of velocity.

Remember, both of integral is areas under curve. So Kind of surprising this is true.

Why is it true?

Can approximate total change in \( F \), \( F(b) - F(a) \), by breaking it into smaller intervals:

\[ \Delta F \approx \text{rate of change of } F \cdot \text{time elapsed} \]

\[ \Rightarrow \Delta F \approx F'(t_i) \cdot \Delta t \]

\[ \Rightarrow \int_a^b F'(t) \, dt \]

\[ \approx \sum_{i=0}^{n-1} F'(t_i) \Delta t \]

\[ \approx \int_a^b F'(t) \, dt \]

\[ \therefore \quad \int_a^b F'(t) \, dt \]

A proof has to deal with lots of details, like estimating errors, and show it's going to zero. Not important in this class. It works.

Why is this important:

1. It gives a way to calculate. Can you find a function \( F \) given \( F' \)? If you can, easy to evaluate!

2. Gives us an interpretation/ use in real life.

the acceleration \( \Rightarrow \) velocity \( \Rightarrow \) position was why Newton invented it.
Q2: Calculate \( \int_0^1 3x^2 \, dx \) exactly. (What function has derivative \( 3x^2 \)?)

\[ f'(x) = 3x^2 \quad \text{then} \quad f(x) = x^3. \]

So, by FTC: \( \int_0^1 3x^2 \, dx = f(1) - f(0) = 1^3 - 0^3 = 1. \)

Of course \( f(x) \) could be \( x^2+1 \) also. (Same deriv)

Q3: If we instead use \( f(x) = x^2+1 \), does what is \( \int_0^1 3x^2 \, dx \) ?

Same! (Good thing too!) \( f(1) - f(0) = (1^3+1) - (0^3+1) = 1+1-0-1 = 1 \checkmark \)

When you find the "anti-derivative," there are always choices. Can add any number.

But \( f(b) - f(a) \), the added numbers cancel out.

Q4: About when does the tortoise pass the hare?

\[ \text{Hare went 7.5 units} \]
\[ \text{So tortoise passes at 7.5 min.} \]

Q5: Who wins?

They tied! Area under both is same: 10.

Also: area between = 0.

Q6: Given graph, when is population the biggest? (where it crosses zero, b/c then it's biggest)

Also, recall why: biggest when slope (deriv) = 0.

Q7: When is growth rate the biggest?
Day 5: 5.4 part I.

We've talked about integrals and what they mean. Today and Friday we'll talk about how to manipulate them.

Could memorize, but wrong way to learn; especially, the first ones make sense in terms of area, so if you remember that, you should remember them.

I'm going to test you along the way before I tell you the rules. O.K.

Q1: \[ \int_0^2 e^{2x}dx + \int_2^4 e^{2x}dx = ? \]
   O.K. Why?

Rule: \[ \int_a^b f(x)dx = -\int_b^a f(x)dx. \] (Switching order switches sign.)

Why? The book says, by definition, \[ \int_a^b f(x)dx = \sum f(x_i)\Delta x, \] but \[ \Delta x = \frac{b-a}{n}. \] So, if switch \( b \) and \( a \), get the minus.

Better: FTC: \[ \int_a^b f(x)dx = F(b)-F(a) = - \int_b^a f(x)dx = -(F(b)-F(a)) = F(b)-F(a). \] Wow!

Why? integral is total change, \( \sum \) ups and downs: total change from \( t=a \) to \( t=b \) is minus the change from \( t=b \) to \( t=a \).

Rule: \[ \int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx \] (note, not adding \( f's \) together!)

Q2: Explain to your neighbor why!
   Ex: Change from \( a \) to \( b \) plus change from \( b \) to \( c \) is change from \( a \) to \( c \).
   Ex:

Q3: Simplify \[ \int_0^1 f(x)dx + \int_0^{-3} f(x)dx. \]
   Rule still works for weird cases.
   Left with this:
   \[ \int_0^c f(x)dx \]
   constant b/c going from \( 1 \) to \( -3 \);
   is, other direction.

Rule: \[ \int_a^b (f(x) + g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx \] Can split integrals over addition.

So adding up \( (f+g)dx \) is same as adding up \( f dx \) and \( g dx \).

Rule: \[ \int_a^b c f(x)dx = c \int_a^b f(x)dx. \] Can pull out constants.

Q4: Explain to neighbor (in terms of graph)
   \[ \int_a^b 2f \]
   twice the height gives twice the area.
   Ex: \[ \int_a^b (3x^2 + \cos x) = 3\int_a^b x^2 dx + \int_a^b \cos xdx \]. So can split into easier pieces.
Q5: Which of these is equivalent to $\int_{1}^{3} f(x) \, dx + \int_{2}^{5} f(x) \, dx$?

None! The combining of limits only works/makes sense if same integrand.

The combining of integrands only works if same limits.

If both are different, can't combine.

Could write as $\int_{1}^{2} f(x) \, dx + \int_{3}^{5} f(x) \, dx$. (Figure out why.)

Integrals are useful for finding all sorts of things, geometrically. If you can break your shape into rectangles, you can get area.

ex: Find area

\[
\begin{array}{c}
\text{height is } f(x) - g(x). \\
\text{break area into rectangles; width is } \Delta x.
\end{array}
\]

So area is $\int_{a}^{b} (f(x) - g(x)) \, dx$! (What is $a$, $b$? When $g(x) = f(x)$. Solve that!)

Can do this kind of trick (find area/volume, add up) lots of ways, but that's 8.1, 8.2.

Q6: Find the area between $f(x) = x^2$ and $g(x) = 1$.

$x^2 = 1$, $x = \pm 1$ is intersections. $1$ is bigger, so

area is $\int_{-1}^{1} (1 - x^2) \, dx = \left[ x - \frac{x^3}{3} \right]_{-1}^{1} = \left( 1 - \frac{1}{3} \right) - \left( -1 - \frac{-1}{3} \right)$

$= \left( 1 - \frac{1}{3} \right) - (-1 + \frac{1}{3})$

$= \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$
Day 6: 5.4 - Part 2.

Sometimes can use symmetry to simplify integrals. Already did me earlier! \( \int_{0}^{\pi} \sin(x) \, dx = 0 \)

cancel each other out.

If \( f \) is even, then \( \int_{-a}^{a} f(x) \, dx = 2 \int_{0}^{a} f(x) \, dx \)

\[ \text{even: } f(-x) = f(x) \]
\[ \text{odd: } f(-x) = -f(x) \]

If \( g \) is odd, then \( \int_{-a}^{a} g(x) \, dx = 0. \) Obvious, but boring.

Really, the even one only moderately useful, since each takes the same amount of work, give or take.

The main example is from statistics: \( f(x) \) bell-shaped.

Q1: The area under the bell curve gives the probability you are within \( \pm a \) standard deviations of the average. However, tables usually only give the area between 0 and \( a \).

So, if the table says \( \int_{0}^{a} f(x) \, dx = 0.3413 \), what is probability of being within 1 standard deviation of the average?

Comparing Integrals: bigger functions have bigger integrals, woah!

This is obvious, but important for integrals with limits going to \( \infty \), etc.

Also, for easy estimates.

So Rule: if \( m \leq f(x) \leq M \) for \( a \leq x \leq b \), then \( m(b-a) \leq \int_{a}^{b} f(x) \, dx \leq M(b-a) \)

if \( f(x) \leq g(x) \) for \( a \leq x \leq b \), then \( \int_{a}^{b} f(x) \, dx \leq \int_{a}^{b} g(x) \, dx \).

Q2: Explain to neighbor the first one. → draw

Q3: T/F: \( \int_{0}^{1} (x+1) \, dx \leq \int_{0}^{1} (x+7) \, dx \)

F: If calculated precisely, -7.5 and -25.5

What went wrong? Went wrong \( \text{way.} \) Rule only works if limits are in the normal order.

Ex: Estimate \( \int_{0}^{1} \sqrt{1 + x^2} \, dx \). Could use Riemann sums, but those are messy.

Know \( 1 \leq \int_{0}^{1} \sqrt{1 + x^2} \, dx \leq \sqrt{2} \) on that interval, so

\( 1.1 \leq \int_{0}^{1} \sqrt{1 + x^2} \, dx \leq 1.4 \)

which is as good as a Riemann sum with a few rects.

A good estimate might be their average, \( \approx 1.2 \).

True value is \( \approx 1.11 \).

Ex: \( \int_{0}^{\infty} \frac{1}{x^2 + 3x + 7} \, dx \) is hard. But \( \int_{0}^{\infty} \frac{1}{x^4} \, dx \), which is easy.
Averages of a function.
This seems weird, but I promise it actually makes sense.

Q4: You start 100 km from your house and travel directly away till you are 300 km away, 2 hours later. What was your average speed?

\[
\text{avg speed} = \frac{\Delta \text{position}}{\Delta \text{time}} = \frac{300 - 100}{2 \text{ hrs}} = 100 \text{ km/hr}.
\]

But you didn’t travel that speed, of course. Speed changed, call speed \( v(t) \).

FTC: \( \Delta \text{position} = P(2) - P(0) = \int_0^2 v(t) \, dt \)

\[
\text{avg speed} = \frac{\Delta \text{position}}{\Delta \text{time}} = \frac{P(2) - P(0)}{2 - 0} = \frac{1}{2 - 0} \int_0^2 v(t) \, dt.
\]

Rule: Average value of \( f(x) \) is \( \frac{1}{b-a} \int_a^b f(x) \, dx = \frac{\Delta F}{\Delta \text{mont}} = \text{average rate of change of F} \).

This seems unconnected to how we normally calculate, but actually same.

avg of \( 3.45 \) is \( \frac{3 + 4 + 5}{3} \leftrightarrow \text{total “width”} \)

Q5: Suppose \( g(t) \) represents the production of gold from a mine, in Kg/day.

Explain what \( \int_0^{365} g(t) \, dt \) and \( \frac{1}{365} \int_0^{365} g(t) \, dt \) represent, including units.

\( \int_0^{365} g(t) \, dt \) has units \( \text{Kg/day} \) \( \times \text{day} = \text{Kg} \), so represents total production in a year.

\( \frac{1}{365} \int_0^{365} g(t) \, dt \) has units \( \text{Kg/day, avg. production/day} \)

\( \text{avg is height so that area under line = area under curve.} \)
If \( F' = f \), we say \( F \) is an antiderivative of \( f \).

ex: \( x^3 \) is an antideriv of \( 3x^2 \), since \( (x^3)' = 3x^2 \).

Similarly, \( x^3 + 3 \) and \( x^3 - 1 \) are also.

We talked about this before, but why?

\( f \) is rate of change, like speed. But speed doesn't care about position, just change in position.

6.2 We'll start calculating some easy ones analytically. Today, graphically.

(Q1) Where is \( F(x) \) positive?

Can't tell! Again, this graph tells us slope, not value.

(Q2) Where increasing? (100)

(Q3) Where concave up? (Everywhere)

(Q4) Does \( F(x) \) have a max or min? Where?

Max/min occur where deriv = 0, so at \( x = 1 \). Decr before, incr after.

So min at \( x = 1 \).

(Q5) Graph an antideriv of

(Q6) For that example, are we 100% sure \( F(2) = F(0) \)? (That is what I drew) T/F.

Once you answer, explain to your neighbor why or why not.

Recall FTC: "area under curve gives total change."

\[
\int_a^b f(x) \, dx = F(b) - F(a).
\]

So what is the total change from \( x = 0 \) to \( x = 2 \)?

\[
\int_0^2 f(x) \, dx = F(2) - F(0)
\]

But this is 0 since symmetric and cancel out.

So \( 0 = F(2) - F(0) \)

\( F(2) = F(0) \).
So, this is pretty cool. Before, we could only roughly estimate the antideriv.
This gives us a way to calculate it.

Q7: What is $F(1)$? (Same ex.) (if $F(0) = 0$) (What is its max,?)

$$\int_0^1 f(x) \, dx = F(1) - F(0) = F(1) - 0 = F(1)$$

Area under b/w 0 and 1, triangle, so \( \frac{1}{2} \) base height = \( \frac{1}{2} \times 1 \times 2 = 1 \)

So $F(1) = 1$.

Q8: Make a table of values for $F(x)$, for $f(x)$ given below: (half-circles)

For $x = 0, 1, 2, 3, 4$.

If $F(0) = \frac{\pi}{2}$.

Area of quarter circle is $\pi r^2 / 2 = \frac{\pi}{4}$.

$$\begin{array}{c|c|c|c|c|c}
   x & 0 & 1 & 2 & 3 & 4 \\
   \hline
   F(x) & -\frac{\pi}{2} & -\frac{\pi}{4} & 0 & \frac{\pi}{4} & -\frac{\pi}{2}
\end{array}$$
Today is pretty straightforward. How to calculate easy integrals.

First: We know derivative of a constant is zero. Converse is true.

Rule: If \( f(x) = 0 \), its antiderivative \( F(x) = C \) for some constant \( C \).

Seems obvious. If rate of change = 0, then original is constant.
One of those obvious things that are annoying to prove.

Before, we said \( f(x) = 3x^2 \) could have antideriv \( x^3 \) or \( x^3 + 1 \) or \( x^3 - 7 \).

Are these all possible antiderivatives?
Suppose \( F(x) \) and \( G(x) \) were both antiderivatives of \( f(x) \).

Then \( F' = G' = f(x) \)

So \( (F - G)' = 0 \), so by prev., \( F - G = C \), so \( F = G + C \).

In other words: any antiderivative only differ by a constant.

Rule: If \( F(x) \) is an antiderivative, any of all other antiderivatives are just \( F(x) + C \).

Indefinite Integral: Notation for antiderivative.

\[
\int f(x) \, dx = F(x) + C
\]

Note: \( \int_a^b f(x) \, dx \)

This is a number.

This gives the antiderivative a function.

Rest of lecture is basic antideriv.

\( f(x) = \text{m} \Rightarrow \text{slope is constant m} \). What is constant slope? a line!

\[
\int \text{m} \, dx = mx + C
\]

We've done \( 2x \Rightarrow x^2 \)
\( 3x^2 \Rightarrow x^3 \) previously. General rule is

\[
\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \text{(add one, divide by new exponent).}
\]

This is similar but opposite to derivative: \( \frac{d}{dx}(x^n) = n \cdot x^{n-1} \).

But note this only works if \( n \neq -1 \).

Q1: What is \( \int x^0 \, dx \)? for \( x > 0 \).

Since \( (\ln x)' = \frac{1}{x} \).
It is kind of weird that \( \frac{1}{x} \) is so special, but it is.

Of course, what about negative \( x \)?

\( (\ln(-x))' = \frac{1}{-x} \cdot -1 = \frac{1}{x} \) so \( \ln(-x) \) is antiderivative for \( x < 0 \).

Combine as \( \int \frac{1}{x} \, dx = \ln|x| + C \).
Since \((e^x)' = e^x\), \(\int e^x \, dx = e^x + C\)

\((\sin x)' = \cos x\), \((\cos x)' = -\sin x\)

so \(\int \cos x \, dx = \sin x + C\), \(\int \sin x \, dx = -\cos x + C\).

Now we have the basic ones.

Unfortunately, things like product/quotient/chain rule don’t really work. General ones are much harder. That’s ch 7. Now, practice:

Q2: Calculate \(\int (3 \cos x - 18 \sqrt{x}) \, dx\)

\[= -3\int \cos x \, dx - 18 \int x^{-1/2} \, dx = -3\sin x - 18 \cdot \frac{x^{1/2}}{1/2} + C = -3\sin x - 12x^{3/2} + C\]

Q3: Find area between \(f(x) = \frac{1}{x}\) and \(g(x) = 1\) between \(x = 1\) and \(x = e\).

\[\text{Area} = \int_1^e (1 - \frac{1}{x}) \, dx = (x - \ln|x| + C)\bigg|_1^e = (e - \ln(e)) - (1 - \ln 1) = e - 1 + 0 = e - 1\]

Q4: Speed of space X rocket is about \(s(t) = 0.09t^2\) m/s. Assuming this, how far does it travel in first 160 s?

\[\int_0^{160} 0.09t^2 \, dt = \left[\frac{0.09}{3} t^3\right]_0^{160} = 0.03 \cdot 160^3 = 122.880 \text{ m} \approx 122.9 \text{ km}\]

\(0.09t^3 + C\) is a function representing total distance traveled.