Math 10B - Discussion 6 Solutions

We know that $0 \leq f(x) \leq g(x)$. If $\int_1^\infty g(x) \, dx$ converges, then the area under the curve of $g(x)$ over the region $[1, \infty)$ is equal to some constant $c$. Since $f(x)$ lies between $g(x)$ and 0, we know that the area under the curve of $f(x)$ over this interval must be less than (or equal to) the area beneath $g(x)$, i.e., $\int_1^\infty f(x) \, dx \leq c$. Similarly, the area under the curve of $f(x)$ must be greater than (or equal to) the area under the 0 function, so $\int_1^\infty f(x) \, dx \geq 0$. Thus, the integral is bounded by two finite values, making it also finite, which proves that it converges.

However, if we only instead that $\int_1^\infty f(x) \, dx$ converges to some constant $c$, then all we know is that the area under the curve of $g(x)$ must be greater than some finite value, i.e., $\int_1^\infty g(x) \, dx \geq c$. Since we have no requirement on the largest value this integral can take, the integral can equal infinity, i.e., the integral can diverge.