Math 10B - Discussion 2 Solutions

The Fundamental Theorem of Calculus states that for a function $f$ that is real and continuous on $[a, b]$ with anti-derivative $F$ on $[a, b]$,

$$\int_{a}^{b} f(x)\,dx = F(b) - F(a).$$

From class, we know that the integral of velocity yields the change in position. Let $f(t)$ be the velocity and $F(t)$ be the position from $t = a$ to $t = b$. Then, we know that the integral $\int_{a}^{b} f(t)\,dt$ is equal to the change in position $\Delta F(t)$, which can be determined by looking at the change from the initial position to the ending position. This change can be represented as follows:

$$\Delta F(t) = F(b) - F(a),$$

i.e., the final position minus the initial position. Therefore, we may conclude:

$$\int_{a}^{b} f(t)\,dt = \Delta F(t)$$

$$= F(b) - F(a)$$

which is precisely what the Fundamental Theorem of Calculus says.