Math 10A Discussion 7 Solutions

1. Let \( f(x) = \sqrt{x} \) and \( g(x) = e^x + 1 \), so that our function is \( f(g(x)) \). To find its derivative, use the chain rule:

\[
(f(g(x)))' = f'(g(x))g'(x) = \frac{1}{2\sqrt{e^x + 1}} \cdot e^x
\]

2. Once again, we’re going to use the chain rule. Let \( f(x) = e^x \), and let \( g(x) = \sqrt{x}e^x \). The derivative of \( f(x) \) is \( f'(x) = e^x \), and the derivative of \( g(x) \) is \( g'(x) = \sqrt{x}e^x + \frac{e^x}{2\sqrt{x}} \) (use the product rule). Our function is \( f(g(x)) \), and we can find its derivative using the chain rule:

\[
(f(g(x)))' = f'(g(x))g'(x) = e^{\sqrt{x}e^x} \cdot \left[ \sqrt{x}e^x + \frac{e^x}{2\sqrt{x}} \right].
\]

3. Volume is measured in \( \text{km}^3 \), and here, the volume \( V(t) \) is a function of time, so \( V'(t) \) should be measured in \( \text{km}^3/\text{year} \). We can show this using the product rule. Since \( V(t) = A(t)T(t) \), we can write

\[
V'(t) = A(t)T'(t) + A'(t)T(t).
\]

\( A(t) \) and \( T(t) \) are both functions of time. The units of \( A(t) \) are \( \text{km}^2 \), so the units of \( A'(t) \) are \( \text{km}^2/\text{year} \). Similarly, the units of \( T(t) \) are \( \text{km} \), so the units of \( T'(t) \) are \( \text{km/year} \). Then, we multiply to get the units of \( A(t)T'(t) \) and \( A'(t)T(t) \), which are
Units of $A(t)T'(t) = (\text{km}^2) \cdot (\text{km/year}) = \text{km}^3/\text{year}$

Units of $A'(t)T(t) = (\text{km}^2/\text{year}) \cdot (\text{km}) = \text{km}^3/\text{year}$.

Now, the units of $V'(t)$ are not $2\text{km}^3/\text{year}$, even though it may look like the units are $\text{km}^3/\text{year} + \text{km}^3/\text{year}$, but we don’t add units. $A(t)T'(t)$ will give us a number in $\text{km}^3/\text{year}$ and $A'(t)T(t)$ will give us a number in $\text{km}^3/\text{year}$, and when we add them to get $V'(t)$, we get an answer in $\text{km}^3/\text{year}$, so the units are just that. One way to think about why the product rule gives us the correct answer is that the volume can change in two ways: it can change in thickness, and it can change in area. The product rule accounts for the fact that these $A$ and $T$ can change independently of each other.

The units of $A'(t)T'(t)$ are $(\text{km}^2/\text{year}) \cdot (\text{km/year}) = \text{km}^3/\text{year}^2$. This however, is not a rate of change of volume with respect to time, so these units are not correct. In fact, this is part of the measurement of the rate of change of the rate of change with respect to time, with respect to time. A simpler way to put it is that this is part of the second derivative of $V(t)$, and we can actually see that if we take its second derivative (using the product rule again):

$$V''(t) = (V'(t))' = (A(t)T'(t) + A'(t)T(t))' = A(t)T''(t) + 2A'(t)T'(t) + A''(t)T(t).$$

Another way to see why this is incorrect is that if one of $A'(t)$ or $T'(t)$ is 0, then $A'(t)T'(t) = 0$. So, for example, if the area doesn’t change but the thickness does, the volume is still changing, even though $A'(t)T'(t) = 0$, so this cannot be the derivative of $V(t)$. 