Homework 4: Due Feb 3, 2016

1. Let $A$ be the linear map from degree 2 polynomials (with the variable $x$) to degree 4 polynomials given by multiplying the polynomial by $x^2 - 1$. Choose a basis for these vector spaces. What matrix represents the linear map $A$, in your chosen basis? What are the dimensions of the column space, kernel, row space and cokernel? Find a basis for the cokernel of $A$.

2. If $Ax = b$ always has at least one solution, prove that the only solution to $A^T y = 0$ is $y = 0$. (Hint: What is the rank?)

3. Explain why the column space of a matrix $A$ and the kernel of the transpose $A^T$ are orthogonal. Also, explain why, if both are subspaces of $\mathbb{R}^m$, that the dimension of the kernel of $A^T$ is $m - r$, where $r$ is the rank of the matrix.