1. Which of the following are subspaces of $\mathbb{R}^\infty$? Explain why or why not.
   (a) All sequences that include infinitely many zeroes.
   (b) All sequences $(x_1, x_2, \cdots)$ with $x_j = 0$ from some point onward. (The $j$ might vary from sequence to sequence.)
   (c) All arithmetic progressions: $x_{j+1} - x_j$ is the same for all $j$.
   (d) All geometric progressions $(x_1, kx_1, k^2x_1, \cdots)$ allowing all $k$ and $x_1$.

2. Explanation question similar to 2.1.5 in the book: (part c from the book is good too, but you don’t have to do it. Notice that my question requires more explanation than the problem from the book.)
   (a) Suppose addition in $\mathbb{R}^2$ adds an extra 1 to each component, so that $(3, 1) + (5, 0)$ is $(9, 2)$ instead of $(8, 1)$. If scalar multiplication is unchanged, which rules of a vector space are broken? Explain why they are broken with sentences and an example.
   (b) Explain why the set of all positive real numbers, with $x + y$ and $cx$ redefined to equal the usual $xy$ and $x^c$, is a vector space. You don’t need to explicitly check each of the 8 properties, but do explain why ”linear combinations” stay in the space. Also, explain: What is the “zero vector”? For an vector $x$, what is the “additive inverse”, $-x$?

3. Prove that the $LDU$ decomposition is unique. (Hint: Assuming you have two decompositions, derive the equation $L_1^{-1}L_2D_2 = D_1U_1U_2^{-1}$. What does this tell you?)