Homework 1: Due Jan 13, 2016

1. Calculate the P, L, D and U of the LDU decomposition for the matrix A below. Then, solve Ax=b, for the b below.

\[
A = \begin{bmatrix}
1 & 2 & 3 & 4 \\
2 & 4 & 3 & 5 \\
3 & 4 & 5 & 6 \\
-2 & -3 & 1 & 1
\end{bmatrix}, \quad b = \begin{bmatrix}
-3 \\
-6 \\
-3 \\
1
\end{bmatrix}
\]

2. Explain how Gaussian elimination can fail, if we do not include row exchanges. Your explanation should include examples, and should discuss when GE can be fixed using row exchanges, and when it cannot. Explain why row exchanges are an acceptable change to make. Explain why, at least for a simple example, if GE cannot be fixed by row exchanges, that the system has either no solutions or infinitely many solutions. (Your explanation should be good enough so that if someone who knew GE, but not about when it fails, read it, they would understand the points mentioned.)

3. Prove that it is impossible for a system of linear equations to have exactly two solutions.

For your information, I will post solutions to these questions after you turn them in. In particular, this should help you understand the quality of explanation I expect for the second and third questions. TA’s will likely produce the solutions for future homeworks.