To: Mathematics Department
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Subject: Numerical Analysis Qualifying Exam Syllabus
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Numerical Analysis Qualifying Exam

The Numerical Analysis Qualifying Exam is based in the syllabus for Mathematics 270ABC. This year the course was divided into six segments, each approximately five weeks in length, that survey major topics in Numerical Analysis and Scientific Computation. These basic topics are:

- Numerical Linear Algebra for Dense Matrices (270A)
- Numerical Linear Algebra for Sparse Matrices (270A)
- Nonlinear Equations and Optimization (270B)
- Approximation Theory (270B)
- Initial Value Problems for Ordinary Differential Equations (270C)
- Two Point Boundary Value Problems for Ordinary Differential Equations (270C)

Because of the timing of the qualifying exam, questions on the last topic (2-point BVP) will not be included in the exam.

In five weeks time, none of these topics can be addressed very deeply. However, those choosing to specialize in some area of numerical analysis or scientific computation need to have some understanding of the issues, both theoretical and algorithmic, in all of these areas. There exist more specialized and advanced courses (e.g., Mathematics 271ABC and Mathematics 272ABC) that address some of these topics in a more comprehensive fashion. The textbook for Mathematics 270ABC, Numerical Mathematics, by Quarteroni, Sacco, and Saleri [1] has chapters that cover most of the material in the syllabus. This textbook is also available online through SpringerLink at http://www.springerlink.com/content/q67046/.

Other texts that provide a survey of many of the topics covered in this course include Dalquist and Björck [2, 3], Atkinson [4], and Isaacson and Keller [5] (a true classic).

References

270A: Numerical Linear Algebra

Numerical Linear Algebra for Dense Matrices

1. **Gaussian Elimination**: Condition number and stability of linear systems of equations, \(LU\) factorization and its variants, pivoting, iterative refinement, roundoff error analysis, complexity.

2. **Linear Least Squares Problems**: basic theory (projection, best approximation), over and under-determined systems, Gram-Schmidt and its variants, Householder and Givens transformations, \(QR\) factorization, singular value decomposition (SVD), stability and conditioning of least squares problems.


Numerical Linear Algebra for Sparse Matrices

1. **Sparse Gaussian Elimination**: sparse matrix data structures, graph model for Gaussian Elimination, computing the fill-in, ordering (minimum degree, nested dissection), complexity, band matrices.

2. **Iterative Methods for Linear Equations**: convergence theory, basic iterative methods (Jacobi, Gauss-Seidel, SOR, SSOR, \(ILU\)), complexity, orderings, convergence criteria.

3. **Krylov Subspace Methods**: Conjugate gradients, preconditioning, generalized condition number, Lanczos method, GMRES, Biconjugate gradients.

Some specialized references on the topics of Mathematics 270A are Golub and Van Loan [1], Stewart [2, 3], Wilkinson [4], Varga [5], and George and Liu [6].

References


270B: Numerical Approximation and Nonlinear Equations

Nonlinear Equations and Optimization


2. *Nonlinear Systems*: Steepest descent, Approximate Newton Methods, Kantorovich Theorem, Quasi Newton Methods, line search, trust region.

3. *Optimization*: unconstrained, equality and inequality constraints, necessary and sufficient conditions, penalty and barrier methods, active set methods, interior point methods, KKT systems, linear and quadratic programming.

Approximation Theory

1. *Polynomial Interpolation*: Lagrange Interpolation, divided differences, Aiken-Neville and Barycentric formulas, error estimates, Runge’s example, Chebyshev approximation, orthogonal polynomials, least squares approximation.


3. *Numerical Quadrature*: Newton-Cotes and Gaussian Quadrature, basic and composite formulas, error estimates, singular integrals, Richardson extrapolation, Euler-Maclaurin Formula, Romberg integration, adaptive quadrature.

Some specialized references on the topics of Mathematics 270B are Gill, Murray and Wright [1], Ortega and Rheinboldt [2], Davis [3], de Boor [4], and Atkinson and Han [5].

References


270C: Numerical Ordinary Differential Equations

Initial Value Problems for Ordinary Differential Equations

1. **Theoretical Background:** existence, uniqueness and stability for first order IVP systems, Lipschitz continuity, Gronwall’s Lemma, converting IVP’s to standard form (first order systems).


3. **Multi-Step Methods:** solution of linear difference equations, Adams methods, predictor corrector, backward difference formulas, weak and strong stability, error analysis and orders of convergence.

Two Point Boundary Value Problems for Ordinary Differential Equations

1. **Theoretical Background:** existence, uniqueness and stability for the 2-point BVP, Ritz and Galerkin variational formulations, Lax-Milgram Theorem.

2. **Finite Difference Methods:** derivation of finite difference equations, consistency, stability and error estimates, convection dominated problems.

3. **Finite Element Methods:** basic formulation, stability and error estimates, duality and Nitsche’s trick, reference elements, streamline diffusion Petrov-Galerkin method for convection dominate problems.

Some specialized references on the topics of Mathematics 270C are Eriksson, Estep, Hansbo, and Johnson [1], Gear [2], Hairer, Nørsett, and Wanner [3, 4] Atkinson and Han [5], and Strang and Fix [6].

References


