Department of Mathematics

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To: Mathematics Department
From: Randolph E. Bank
Subject: Numerical Analysis Qualifying Exam Syllabus
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Numerical Analysis Qualifying Exam

The Numerical Analysis Qualifying Exam is based in the syllabus for Mathematics 270ABC. This year the course was divided in to six segments, each approximately five weeks in length, that survey major topics in Numerical Analysis and Scientific Computation. These basic topics are:

- Numerical Linear Algebra for Dense Matrices (270A)
- Numerical Linear Algebra for Sparse Matrices (270A)
- Nonlinear Equations and Optimization (270B)
- Approximation Theory (270B)
- Initial Value Problems for Ordinary Differential Equations (270C)
- Two Point Boundary Value Problems for Ordinary Differential Equations (270C)

Because of the timing of the qualifying exam, questions on the last topic (2-point BVP) will not be included in the exam.

In five weeks time, none of these topics can be addressed very deeply. However, those choosing to specialize in some area of numerical analysis or scientific computation need to have some understanding of the issues, both theoretical and algorithmic, in all of these areas. There exist more specialized and advanced courses (e.g., Mathematics 271ABC and Mathematics 272ABC) that address some of these topics in a more comprehensive fashion. The textbook for Mathematics 270ABC, *Numerical Mathematics*, by Quarteroni, Sacco, and Saleri [1] has chapters that cover most of the material in the syllabus. This textbook is also available online through SpringerLink at http://www.springerlink.com/content/q67046/. Other texts that provide a survey of many of the topics covered in this course include Dalquist and Björck [2, 3], Atkinson [4], and Isaacson and Keller [5] (a true classic).

References

[1] Alfio Quarteroni, Riccardo Sacco, and Fausto Saleri. *Numerical mathematics*, volume 37 of *Texts in Applied Mathematics*. Springer-Verlag, Berlin, second edition, 2007.

[2] Åke Björck and Germund Dahlquist. *Numerical methods*. Prentice-Hall Inc., Englewood Cliffs, N.J., 1974. Translated from the Swedish by Ned Anderson, Prentice-Hall Series in Automatic Computation.

[3] Germund Dahlquist and Åke Björck. *Numerical methods*. Dover Publications Inc., Mineola, NY, 2003. Translated from the Swedish by Ned Anderson, Reprint of the 1974 English translation.

[4] Kendall E. Atkinson. *An introduction to numerical analysis*. John Wiley & Sons Inc., New York, second edition, 1989.

[5] Eugene Isaacson and Herbert Bishop Keller. *Analysis of numerical methods*. Dover Publications Inc., New York, 1994. Corrected reprint of the 1966 original [Wiley, New York; MR0201039 (34 #924)].

270A: Numerical Linear Algebra

Numerical Linear Algebra for Dense Matrices

1. *Gaussian Elimination:* Condition number and stability of linear systems of equations, LU factorization and its variants, pivoting, iterative refinement, roundoff error analysis, complexity.

2. *Linear Least Squares Problems:* basic theory (projection, best approximation), over and under-determined systems, Gram-Schmidt and its variants, Householder and Givens transformations, QR factorization, singular value decomposition (SVD), stability and conditioning of least squares problems.

3. *Algebraic Eigenvalue Problem:* Schur Decomposition, power method, inverse iteration, Rayleigh Quotient iteration, Hessenberg form, QR method for eigenvalues, Sturm sequences, Gerschgorin estimates, conditioning of eigenvalues and eigenvectors.

Numerical Linear Algebra for Sparse Matrices

1. *Sparse Gaussian Elimination:* sparse matrix data structures, graph model for Gaussian Elimination, computing the fill-in, ordering (minimum degree, nested dissection), complexity, band matrices.

2. *Iterative Methods for Linear Equations:* convergence theory, basic iterative methods (Jacobi, Gauss-Seidel, SOR, SSOR, ILU), complexity, orderings, convergence criteria.

3. *Krylov Subspace Methods:* Conjugate gradients, preconditioning, generalized condition number, Lanczos method, GMRES, Biconjugate gradients.

Some specialized references on the topics of Mathematics 270A are Golub and Van Loan [1], Stewart [2, 3], Wilkinson [4], Varga [5], and George and Liu [6].

References

[1] Gene H. Golub and Charles F. Van Loan. *Matrix computations.* Johns Hopkins Studies in the Mathematical Sciences. Johns Hopkins University Press, Baltimore, MD, third edition, 1996.

[2] G. W. Stewart. *Matrix algorithms. Vol. I.* Society for Industrial and Applied Mathematics, Philadelphia, PA, 1998. Basic decompositions.

[3] G. W. Stewart. *Matrix algorithms. Vol. II.* Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2001. Eigensystems.

[4] J. H. Wilkinson. *The algebraic eigenvalue problem.* Monographs on Numerical Analysis. The Clarendon Press Oxford University Press, New York, 1988. Oxford Science Publications.

[5] Richard S. Varga. *Matrix iterative analysis*, volume 27 of *Springer Series in Computational Mathematics.* Springer-Verlag, Berlin, expanded edition, 2000.

[6] Alan George and Joseph W. H. Liu. *Computer solution of large sparse positive definite systems.* Prentice-Hall Inc., Englewood Cliffs, N.J., 1981. Prentice-Hall Series in Computational Mathematics.

270B: Numerical Approximation and Nonlinear Equations

Nonlinear Equations and Optimization

1. *Scalar Equations:* Fixed point iteration, bisection, Newton's method, secant method, order of convergence.

2. *Nonlinear Systems:* Steepest descent, Approximate Newton Methods, Kantorovich Theorem, Quasi Newton Methods, line search, trust region.

3. *Optimization:* unconstrained, equality and inequality constraints, necessary and sufficient conditions, penalty and barrier methods, active set methods, interior point methods, KKT systems, linear and quadratic programming.

Approximation Theory

1. *Polynomial Interpolation:* Lagrange Interpolation, divided differences, Aiken-Neville and Barycentric formulas, error estimates, Runge's example, Chebyshev approximation, orthogonal polynomials, least squares approximation.

2. *Piecewise Polynomial Approximation:* C^0 approximation spaces, error estimates for interpolation and least squares, Peano Kernel Theorem, Hermite and spline approximation.

3. *Numerical Quadrature:* Newton-Cotes and Gaussian Quadrature, basic and composite formulas, error estimates, singular integrals, Richardson extrapolation, Euler-Maclaurin Formula, Romberg integration, adaptive quadrature.

Some specialized references on the topics of Mathematics 270B are Gill, Murray and Wright [1], Ortega and Rheinboldt [2], Davis [3], de Boor [4], and Atkinson and Han [5].

References

[1] Philip E. Gill, Walter Murray, and Margaret H. Wright. *Practical optimization.* Academic Press Inc. [Harcourt Brace Jovanovich Publishers], London, 1981.

[2] J. M. Ortega and W. C. Rheinboldt. *Iterative solution of nonlinear equations in several variables*, volume 30 of *Classics in Applied Mathematics.* Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2000. Reprint of the 1970 original.

[3] Philip J. Davis. *Interpolation and approximation.* Dover Publications Inc., New York, 1975. Republication, with minor corrections, of the 1963 original, with a new preface and bibliography.

[4] Carl de Boor. *A practical guide to splines*, volume 27 of *Applied Mathematical Sciences.* Springer-Verlag, New York, revised edition, 2001.

[5] Kendall Atkinson and Weimin Han. *Theoretical numerical analysis*, volume 39 of *Texts in Applied Mathematics.* Springer, Dordrecht, third edition, 2009. A functional analysis framework.

270C: Numerical Ordinary Differential Equations

Initial Value Problems for Ordinary Differential Equations

1. *Theoretical Background:* existence, uniqueness and stability for first order IVP systems, Lipschitz continuity, Gronwall's Lemma, converting IVP's to standard form (first order systems).

2. *Single-Step Methods:* explicit and diagonally implicit Runge-Kutta and Runge-Kutta-Fehlberg methods, error estimates and order of convergence, absolute stability, A and L stability, stiff equations, adaptive time stepping.

3. *Multi-Step Methods:* solution of linear difference equations, Adams methods, predictor corrector, backward difference formulas, weak and strong stability, error analysis and orders of convergence.

Two Point Boundary Value Problems for Ordinary Differential Equations

1. *Theoretical Background:* existence, uniqueness and stability for the 2-point BVP, Ritz and Galerkin variational formulations, Lax-Milgram Theorem.

2. *Finite Difference Methods:* derivation of finite difference equations, consistency, stability and error estimates, convection dominated problems.

3. *Finite Element Methods:* basic formulation, stability and error estimates, duality and Nitsche's trick, reference elements, streamline diffusion Petrov-Galerkin method for convection dominate problems.

Some specialized references on the topics of Mathematics 270C are Eriksson, Estep, Hansbo, and Johnson [1], Gear [2], Hairer, Nørsett, and Wanner [3, 4] Atkinson and Han [5], and Strang and Fix [6].

References

[1] K. Eriksson, D. Estep, P. Hansbo, and C. Johnson. *Computational differential equations.* Cambridge University Press, Cambridge, 1996.

[2] C. William Gear. *Numerical initial value problems in ordinary differential equations.* Prentice-Hall Inc., Englewood Cliffs, N.J., 1971.

[3] E. Hairer, S. P. Nørsett, and G. Wanner. *Solving ordinary differential equations. I*, volume 8 of *Springer Series in Computational Mathematics.* Springer-Verlag, Berlin, second edition, 1993. Nonstiff problems.

[4] E. Hairer and G. Wanner. *Solving ordinary differential equations. II*, volume 14 of *Springer Series in Computational Mathematics.* Springer-Verlag, Berlin, 1991. Stiff and differential-algebraic problems.

[5] Kendall Atkinson and Weimin Han. *Theoretical numerical analysis*, volume 39 of *Texts in Applied Mathematics.* Springer, Dordrecht, third edition, 2009. A functional analysis framework.

[6] Gilbert Strang and George J. Fix. *An analysis of the finite element method.* Prentice-Hall Inc., Englewood Cliffs, N. J., 1973. Prentice-Hall Series in Automatic Computation.