MATH 270C: Numerical Ordinary Differential Equations

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Spring Quarter 2018

Homework Assignment #5
Due Friday, May 11, 2018

Exercise 5.1. Let $y' = f(y)$, $y(0) = y_0$, $x_k = kh$, $k = 0, 1, \ldots$, where $h > 0$ is fixed. Let $p \geq 0$, $r \geq 0$ be integers. From

$$\int_{x_{k-p}}^{x_{k+1}} y' \, dx = \int_{x_{k-p}}^{x_{k+1}} f(y) \, dx$$

we get

$$y(x_{k+1}) = y(x_{k-p}) + \int_{x_{k-p}}^{x_{k+1}} f(y) \, dx$$

A multistep method is obtained by interpolating f at $x_{k+1}, x_k, \ldots, x_{k-r+1}$ by a polynomial of degree r, and then integrating that interpolating polynomial exactly.

a. Prove that any multistep method derived in this fashion is *consistent*.

b. Prove the scheme is *stable* is $p = 0$ and *weakly stable* is $p > 0$.

Exercise 5.2. Consider the initial value problem:

$$\begin{aligned} y' &= f(y) \\ y(x_0) &= y_0 \end{aligned}$$

and the consider the second backward difference formula:

$$y_{k+1} = \alpha_1 y_k + \alpha_2 y_{k-1} + h\beta_0 f(y_{k+1})$$

a. Find α_1, α_2 and β_0 to maximize the order.

b. Find the local truncation error.

c. Find the region of absolute stability for the method. Is the method A-stable? L-stable?