Math 270c: Numerical Ordinary Differential Equations

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Spring Quarter 2018

Homework Assignment #4
Due Friday, May 4, 2018

Exercise 4.1. **a.** Find the general solution of the linear difference equation

$$u_{n+1} = u_n + \frac{1}{4}(u_{n-1} - u_{n-2})$$

for $n \geq 2$.

b. Find the solution for initial conditions $u_0 = 4$, $u_1 = 3/2$, and $u_2 = 7/4$.

Exercise 4.2. Consider the two-step method

$$y_{n+1} = \frac{1}{2}(y_n + y_{n-1}) + \frac{h}{4}(4f_{n+1} - f_n + 3f_{n-1})$$

for $n \geq 1$ and $y' = f(t, y)$.

a. Show this is a second order method.

b. Find the leading term of the truncation error.

Exercise 4.3. **a.** Find all *explicit* fourth-order formulae of the form

$$y_{n+1} = \alpha_0 y_n + \alpha_1 y_{n-1} + \alpha_2 y_{n-2} + h(\beta_0 f_n + \beta_1 f_{n-1} + \beta_2 f_{n-2})$$

for $n \geq 2$ and $y' = f(t, y)$.

b. Prove that every explicit three-step method of order 4 is unstable.