MATH 270C: Numerical Ordinary Differential Equations

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Spring Quarter 2018

Homework Assignment #3
Due Friday, April 27, 2018

Exercise 3.1. Consider the one stage implicit Runge-Kutta method for $y' = f(y)$:

$$\begin{aligned} z_k &= hf(y_k + \beta z_k) \\ y_{k+1} &= y_k + z_k \end{aligned}$$

a. Find β such that the method is second order accurate.

b. Find the leading term of the local truncation error.

c. Find the region of absolute stability for the method. Is the method A-stable? L-stable?

Exercise 3.2. Consider the formulae:

$$\begin{aligned} y_{k+1/2} &= y_k + \frac{h}{2} f\left(\frac{y_{k+1/2} + y_k}{2}\right) \\ y_{k+1} &= \frac{4}{3} y_{k+1/2} - \frac{1}{3} y_k + \frac{h}{3} f(y_{k+1}) \end{aligned}$$

for $0 < \epsilon \leq 1$. This is a two stage diagonally implicit Runge-Kutta method (in disguise).

a. Put this in the standard form of a diagonally implicit Runge-Kutta method.

$$\begin{aligned} v_1 &= f(y_k + h\beta_{11} v_1) \\ v_2 &= f(y_k + h\beta_{21} v_1 + h\beta_{22} v_2) \\ y_{k+1} &= y_k + h\gamma_1 v_1 + h\gamma_2 v_2 \end{aligned}$$

b. Show the method is second order.

c. Find the region of absolute stability for the method. Is the method A-stable? L-stable?