## MATH 270C: Numerical Ordinary Differential Equations

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Homework Assignment #2 Due Friday, April 20, 2018

Exercise 2.1. Consider the second order initial value problem:

 $y'' + 11y' + 10y = -11\sin x + 9\cos x$ 

with initial conditions y(0) = 1 and y'(0) = 0. The exact solution is  $y = \cos x$ .

- a. Convert this to a system of first order differential equations in standard form.
- **b.** Write a program to solve this system using Euler's method. Use the (time) interval  $0 \le x \le 10$ . Do the calculation three times, using a constant stepsize of h = 1, h = 1/10 and h = 1/100. Perturb the initial data to  $y'(0) = 10^{-6}$  (this should enhance the effect that we are looking for without the necessity of solving a more complicated problem). Since you know the exact solution, you can also compute the error.
- c. Find the general solution to the homogeneous equation

$$y'' + 11y' + 10y = 0$$

**d.** Find the eigenvalues of the  $2 \times 2$  matrix

$$A = \begin{pmatrix} 1 & h \\ -10h & 1 - 11h \end{pmatrix}$$

What relevance does this matrix have to Euler's method for this problem?

e. Using what you learned in parts c and d, explain the observed behavior of the error.