Math 270c: Numerical Ordinary Differential Equations

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Spring Quarter 2018

Homework Assignment #1
Due Friday, April 13, 2018

Exercise 1.1. Prove Gronwall's Lemma: Let

$$y' + \gamma \leq \alpha + \beta y$$

for $0 \leq t \leq T$, and $\alpha, \beta, \gamma, y \geq 0$, β constant. Then

$$\max_{0 \leq t \leq T} y(t) + \int_0^T \gamma \, dt \leq c(\beta, T) \left\{ \int_0^T \alpha \, dt + y(0) \right\}.$$

Exercise 1.2. Prove:

- **a.** $1 + x \leq e^x$ for all $x \in \mathbb{R}$.
- **b.** $(1 + x)^m \leq e^{mx}$ for all $x \geq 0$ and $m = 0, 1, 2 \ldots$.
- **c.** $\left(1 + \frac{y}{m}\right)^m \leq e^y$ for all $y \geq 0$ and $m = 1, 2, 3 \ldots$.

Exercise 1.3. Prove the discrete Gronwall Lemma: Let

$$\frac{y_{k+1} - y_k}{h} + \gamma_k \leq \alpha_k + \beta y_k$$

for $0 \leq k \leq N - 1$, and $\alpha_k, \beta, \gamma_k, y_k \geq 0$, β constant. Then for $T = N * h$,

$$\max_{0 \leq k \leq N} y_k + h \sum_{k=0}^{N-1} \gamma_k \leq c(\beta, T) \left\{ h \sum_{k=0}^{N-1} \alpha_k + y_0 \right\}.$$

You will need to use the previous exercise.