MATH 270B: Numerical Approximation and Nonlinear Equations

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Winter Quarter 2020

Homework Assignment #4
Due Friday, January 31, 2020

Exercise 4.1. The *primal* form of a linear program is:

$$\min_{Ax \geq b} c^t x$$

The *dual* problem is:

$$\max_{A^t y = c,\, y \geq 0} b^t y$$

In this problem we illustrate the connection between primal and dual formulations. We assume A has full rank.

a. Find necessary conditions for the solution of the primal problem. In particular,

$$Ax^* \geq b, \hat{A}x^* = \hat{b}$$
$$c = \hat{A}^t \lambda^*$$
$$\lambda^* \geq 0$$

where x^* is the solution, $\hat{A}$ corresponds to active constraints at the solution and λ^* are the corresponding Lagrange multipliers.

b. We show that y^* given by

$$y^* = \begin{pmatrix} \lambda^* \\ 0 \end{pmatrix}$$

is the solution of the dual problem.

Assume A be can be partitioned as $A^t = (\hat{A}^t \bar{A}^t)$. Verify that y^* is feasible.

$$A^t y^* = c$$
$$y^* \geq 0$$

The active constaints for the dual problem are all of the equality constraints plus the inequality constraints for which equality holds. We may express this as

$$\begin{pmatrix} \hat{A}^t & \bar{A}^t \\ 0 & I \end{pmatrix} \begin{pmatrix} \lambda^* \\ 0 \end{pmatrix} = \begin{pmatrix} c \\ 0 \end{pmatrix}.$$

c. Noting that $\max b^t y = \min -b^t y$, verify that the Lagrange multipliers σ for the dual problem satisfy

$$\begin{pmatrix} \hat{A} & 0 \\ \bar{A} & I \end{pmatrix} \begin{pmatrix} \hat{\sigma} \\ \bar{\sigma} \end{pmatrix} = -b = -\begin{pmatrix} \hat{b} \\ \bar{b} \end{pmatrix}$$

Thus verify

$$\hat{\sigma} = -x^*$$
$$\bar{\sigma} = \bar{A}x^* - \bar{b} > 0$$

d. Summarize the necessary conditions for the solution of the dual problem, and show that they are satisfied by y^* and σ using parts b and c.