MATH 270B: Numerical Approximation and Nonlinear Equations

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Winter Quarter 2020

Homework Assignment #1
Due Friday, January 10, 2020

Exercise 1.1. A function $f(x) = (x - x^*)^m h(x)$, $m \geq 1$, with $h(x^*) \neq 0$ is said to have a root of multiplicity m at x^*.

a. Prove that x^* is a root of $f(x)$ with multiplicity m if and only if $f(x^*) = f'(x^*) = \cdots = f^{m-1}(x^*) = 0$ and $f^m(x^*) \neq 0$.

b. Prove that if $m > 1$, then Newton's method is only linearly convergent.

c. Show that second order convergence is recovered for the modified Newton method

$$x_{k+1} = x_k - \frac{mf(x_k)}{f'(x_k)}.$$

d. Show that second order convergence is recovered for the modified Newton method

$$x_{k+1} = x_k - \frac{f^{m-1}(x_k)}{f^m(x_k)}.$$

e. Consider $g(x) = f(x)/f'(x)$. Show that second order convergence is recovered for the Newton method

$$x_{k+1} = x_k - \frac{g(x_k)}{g'(x_k)}.$$

While these modifications (as well as others) speed convergence for multiple roots, they require one to *know* that there is a multiple root and in some cases to also to know the multiplicity m. This limits their use in many practical applications.

Exercise 1.2. Consider the function $f(x) = (x - \alpha)^m$ for $m \geq 1$. Here we will study the sensitivity of multiple roots in a simple case. Let ϵ be a small perturbation, and let $\hat{f}(x) = f(x) + \epsilon$. Analyze the zero structure of $\hat{f}$, and explain why it is very difficult in general to accurately compute multiple roots in a floating point number system.