

MATH 270A: Numerical Linear Algebra

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Fall Quarter 2017

Homework Assignment #7
Due Wednesday, November 15, 2017

Exercise 7.1. Let A be symmetric, positive definite, with a nonzero structure given by

$$\begin{pmatrix} x & x & & x & & x \\ x & x & & & x & \\ & & x & & x & x \\ x & & & x & & \\ & x & x & & x & \\ x & & x & & & x \end{pmatrix}$$

- Compute the JA/A data structures for A .
- Find the graph of A .
- Compute the sequence of elimination graphs corresponding to the factorization $A = LDL^t$.

Exercise 7.2. A *tree* is an acyclic connected graph. Show the following simple properties of trees:

- The path between any two vertices in a tree is unique.
- A tree with n vertices has exactly $n - 1$ edges.
- A tree has at least one vertex of degree one (called a *leaf*).

Exercise 7.3. Let M be a symmetric, positive definite matrix whose graph is a tree.

- Show the graph of a tridiagonal matrix is a tree.
- Show the graph of an arrow matrix is a tree.
- Prove that any symmetric, positive definite matrix whose graph is a tree can be factored as $PMP^t = LDL^t$ without fill-in.

Exercise 7.4. Let A be an $n \times n$ symmetric, positive definite matrix with nonzeros $a_{i,i}$, $1 \leq i \leq n$, $a_{i,i+1} = a_{i+1,i}$, $1 \leq i \leq n - 1$, and $a_{1,n} = a_{n,1}$ (A is tridiagonal with two extra “corner” elements).

- Find the graph of A .
- Show the fill-in for any ordering of A must be $n - 3$ edges.