MATH 270A: Numerical Linear Algebra

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Fall Quarter 2017

Homework Assignment #5
Due Wednesday, November 1, 2017

Exercise 5.1. Let

$$v = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \qquad \text{and} \qquad w = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

Let $A = vw^t$ be a 3×2 matrix of rank 1.

a. Compute the generalized inverse $A^\dagger$ by computing the generalized inverses of the rank 1 matrices v and w^t.
b. Compute the singular value decomposition $A = Q\Lambda\hat{Q}$.
c. Compute the generalized inverse $A^\dagger$ using the inverses of Q and $\hat{Q}$ and the generalized inverse of Λ.

Exercise 5.2. A *monic* polynomial is one whose highest degree term has coefficient 1. Thus

$$p(\lambda) = \lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_1\lambda + a_0$$

is monic. With the monic polynomial p, we associate the $n \times n$ *companion matrix* A defined by

$$A = \begin{pmatrix} & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{pmatrix}$$

The empty spaces are zeroes. Notice that A has the coefficients of $-p$ in the last row. The diagonal of ones is *not* the main diagonal but rather the elements $A_{k,k+1}$, $1 \le k \le n-1$. Prove (by induction, using expansion by minors) that $det(I\lambda - A) = p(\lambda)$. Thus the zeroes of p are the eigenvalues of A.

Exercise 5.3. Let A be an $N \times N$ real matrix with a complex conjugate pair of eigenvalues $\alpha \pm i\beta$ and corresponding eigenvectors $x \pm iy$. Verify that span$\{x, y\}$ is an invariant subspace of A.

Exercise 5.4. A be and $N \times N$ real matrix. Prove there exists an orthogonal matrix Q such that $A = QTQ^t$, where T is a real block upper triangular matrix with 1×1 and 2×2 diagonal blocks, and each 2×2 block corresponds to a pair of complex conjugate eigenvalues. Hint: use induction. We need 2 base cases: 1×1 and 2×2. The induction step also has two cases. The 1×1 case is similar to the proof given is class. The 2×2 case makes use of the previous exercise.