Math 270a: Numerical Linear Algebra

Instructor: Randolph E. Bank

Fall Quarter 2017

Homework Assignment #2
Due Wednesday, October 11, 2017

Exercise 2.1. Let A be an $n \times n$ nonsingular matrix. Prove $PA = LDU$, where P is a permutation matrix, L is unit lower triangular, D is diagonal and U is *unit* upper triangular.

Exercise 2.2. Let A be an $n \times n$ symmetric, positive definite matrix and consider the partition

$$A = \begin{pmatrix} \alpha & c^t \\ c & B \end{pmatrix},$$

where α is a scalar and B is $n-1 \times n-1$. Prove the $n-1 \times n-1$ matrix

$$S = B - \frac{cc^t}{\alpha}$$

is also symmetric positive definite.

Exercise 2.3. Let A be an $n \times n$ symmetric positive definite matrix. Prove $A = LDL^t$, where L is unit lower triangular and D is diagonal with positive diagonal entries. (Hint: use Exercise 2.2 and induction.)

Exercise 2.4. Let A be an $n \times n$ symmetric positive definite matrix. Prove $A = LL^t$, where L is lower triangular with positive diagonal entries. This factorization is called the Cholesky factorization.

Exercise 2.5. Let A be an $n \times n$ symmetric, positive definite matrix, and consider the partition

$$A = \begin{pmatrix} B & c \\ c^t & \alpha \end{pmatrix},$$

where B is $n-1 \times n-1$ and α is a scalar. Let B have the decomposition $B = \bar{L}\bar{D}\bar{L}^t$. Use this decomposition to compute the LDL^t decomposition of A. This is another variant of Gaussian elimination called *bordering*.