Math 270a: Numerical Linear Algebra

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Fall Quarter 2017

Homework Assignment #10
Due Wednesday, December 6, 2017

Exercise 10.1. This Matlab problem concerns the rate of convergence of the SSOR iteration as a function of both the relaxation parameter ω and the dimension of the problem N. We will solve the tridiagonal system

$$Tx = 0$$

where T is the tridiagonal matrix with $T_{ii} = 2$, $T_{i+1,i} = T_{i-1,i} = -1$. Our initial guess will be $x_0^t = (1, 1, 1, \ldots, 1)$. Since the solution is (trivially) $x = 0$, you will be able to easily compute errors and study the convergence behavior.

Let $T = D - L - L^t$, $r_0 = b - Tx_0$ (note $b = 0$). Program the SSOR method as follows; it is a little cumbersome, but will make the next exercise easier.

$$\begin{aligned}
(D - \omega L)y_k &= \omega(2-\omega)r_{k-1} \\
(D - \omega L^t)z_k &= Dy_k \\
p_k &= z_k \\
s_k &= Tp_k \\
x_k &= x_{k-1} + p_k \\
r_k &= r_{k-1} - s_k
\end{aligned}$$

Iterate until $||x_k|| \le \epsilon$.

a. For $N = 31, 63, 127$, and $\omega = 0.1, 1.0, 1.5, 1.9$ count the number of iterations required to achieve $||x_k|| \le \epsilon = 10^{-4}$. Make a table of your results.

b. What general conclusions can you draw about the behavior of the SSOR method as a function of N, and as a function of ω? Is this consistent with your analysis in exercise 9.1?

c. For $N = 63$, experimentally (i.e., by trial and error) determine the best value for ω. Use exercise 9.1 to help you get started.

Exercise 10.2. This Matlab problem concerns the rate of convergence of the SSOR matrix used as a preconditioner for the conjugate gradient method. We will solve the tridiagonal system

$$Tx = 0$$

where T is the tridiagonal matrix with $T_{ii} = 2$, $T_{i+1,i} = T_{i-1,i} = -1$. Our initial guess will be $x_0^t = (1, 1, 1, \ldots, 1)$. The exact solution is $x = 0$.

Let $T = D - L - L^t$, $r_0 = b - Tx_0$ (note $b = 0$). Program the SSOR-CG method as follows:

$$\begin{aligned}
(D - \omega L)y_k &= \omega(2-\omega)r_{k-1} \\
(D - \omega L^t)z_k &= Dy_k \\
\gamma_k &= z_k^t r_{k-1} \\
\beta_k &= \frac{\gamma_k}{\gamma_{k-1}} \\
p_k &= z_k + \beta_k p_{k-1} \\
s_k &= Tp_k \\
\eta_k &= p_k^t s_k \\
\alpha_k &= \frac{\gamma_k}{\eta_k} \\
x_k &= x_{k-1} + \alpha_k p_k \\
r_k &= r_{k-1} - \alpha_k s_k
\end{aligned}$$

To get the initial conditions right, and to be sure you don't divide by zero, set $p_0 = 0$ and $\gamma_0 = 1$. Iterate until $||x_k|| \le \epsilon$.

a. For $N = 31, 63, 127$, and $\omega = 0.1, 1.0, 1.5, 1.9$ count the number of iterations required to achieve $||x_k|| \le \epsilon = 10^{-4}$. Make a table of your results. Solve also for $N = 63$ and the optimal value of ω you determined in exercise 10.1.

b. In comparing your results with exercise 10.1, what conclusions can you draw about the behavior of the SSOR-CG method in comparison with the ordinary SSOR method? Is the rate of convergence for the SSOR method as sensitive to the choice of ω as the unaccelerated method?