Exercise 10.1. This Matlab problem concerns the rate of convergence of the SSOR iteration as a function of both the relaxation parameter $\omega$ and the dimension of the problem $N$. We will solve the tridiagonal system

$$Tx = 0$$

where $T$ is the tridiagonal matrix with $T_{ii} = 2$, $T_{i+1,i} = T_{i-1,i} = -1$. Our initial guess will be $x_0 = (1, 1, 1, \ldots, 1)$. Since the solution is (trivially) $x = 0$, you will be able to easily compute errors and study the convergence behavior.

Let $T = D - L - L^T$, $r_0 = b - Tx_0$ (note $b = 0$). Program the SSOR method as follows; it is a little cumbersome, but will make the next exercise easier.

$$(D - \omega L)y_k = \omega(2 - \omega)r_{k-1}$$

$$(D - \omega L^T)z_k = Dy_k$$

$p_k = z_k$

$s_k = Tp_k$

$x_k = x_{k-1} + p_k$

$r_k = r_{k-1} - s_k$

Iterate until $||x_k|| \leq \epsilon$.

a. For $N = 31, 63, 127$, and $\omega = 0.1, 1.0, 1.5, 1.9$ count the number of iterations required to achieve $||x_k|| \leq \epsilon = 10^{-4}$. Make a table of your results.

b. What general conclusions can you draw about the behavior of the SSOR method as a function of $N$, and as a function of $\omega$? Is this consistent with your analysis in exercise 9.1?

c. For $N = 63$, experimentally (i.e., by trial and error) determine the best value for $\omega$. Use exercise 9.1 to help you get started.

Exercise 10.2. This Matlab problem concerns the rate of convergence of the SSOR matrix used as a preconditioner for the conjugate gradient method. We will solve the tridiagonal system

$$Tx = 0$$

where $T$ is the tridiagonal matrix with $T_{ii} = 2$, $T_{i+1,i} = T_{i-1,i} = -1$. Our initial guess will be $x_0 = (1, 1, 1, \ldots, 1)$. The exact solution is $x = 0$. 
Let $T = D - L - L^t$, $r_0 = b - Tx_0$ (note $b = 0$). Program the SSOR-CG method as follows:

\[(D - \omega L)y_k = \omega(2 - \omega)r_{k-1}\]
\[(D - \omega L^t)z_k = Dy_k\]
\[\gamma_k = \frac{z_k^t r_{k-1}}{\gamma_{k-1}}\]
\[\beta_k = \frac{\gamma_k}{\gamma_{k-1}}\]
\[p_k = z_k + \beta_k p_{k-1}\]
\[s_k = Tp_k\]
\[\eta_k = \frac{p_k^t s_k}{\eta_k}\]
\[\alpha_k = \frac{\gamma_k}{\eta_k}\]
\[x_k = x_{k-1} + \alpha_k p_k\]
\[r_k = r_{k-1} - \alpha_k s_k\]

To get the initial conditions right, and to be sure you don’t divide by zero, set $p_0 = 0$ and $\gamma_0 = 1$. Iterate until $||x_k|| \leq \epsilon$.

**a.** For $N = 31, 63, 127$, and $\omega = 0.1, 1.0, 1.5, 1.9$ count the number of iterations required to achieve $||x_k|| \leq \epsilon = 10^{-4}$. Make a table of your results. Solve also for $N = 63$ and the optimal value of $\omega$ you determined in exercise 10.1.

**b.** In comparing your results with exercise 10.1, what conclusions can you draw about the behavior of the SSOR-CG method in comparison with the ordinary SSOR method? Is the rate of convergence for the SSOR method as sensitive to the choice of $\omega$ as the unaccelerated method?