Chapter 5

3. Oriented graph

4. $A_1$ has oriented graph
   - No connection from 1 to 2 or 4
   - Not strongly connected, $A_1$ reducible

   $A_2$ has oriented graph which could be represented as permutation

   $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$

   It has order 4, so the oriented graph is strongly connected.

5. Do permutation

   $(2,3)(3,4)(4,5)(1,2) \ A \ (1,2)(4,5)(3,4)\ 1,2,3)$

   $A$ can be reduced to

   $M_1 = \begin{pmatrix} 2 & 1 \\ 0.5 & 4 \end{pmatrix}$

   $R_1 \ R_2$ are separated, thus each contains one eigenvalue.
Since \(A\) is real, the complex eigenvalues must appear as conjugate pairs. So the eigenvalues in \(R_1\) and \(R_2\) must be real.

\[
M_2 = \begin{pmatrix}
-4 & 0 & 0.5 \\
0.5 & -1 & 0 \\
0.5 & 0.2 & 3
\end{pmatrix}
\]

\(R_1, R_2, R_3\) are separated thus each contains one eigenvalue and they must be real.

6. Wrong Problem

Example: \(A = \begin{pmatrix} 1 & 1 \\ -2 & 3 \end{pmatrix}\)

has eigenvalues \(2 \pm i\);

they are both outside \(R_1\).

By the Gershgorin Disk Theorem could not guarantee each disk contains eigenvalue, unless they are separated.
$10. \quad X = \sum_{i=2}^{n} C_i X_i$

$A^k X = \sum_{i=2}^{n} C_i \lambda_i^k X_i = \lambda_2^k \left( C_2 X_2 + C_3 \left( \frac{\lambda_3}{\lambda_2} \right)^k X_3 + \cdots + C_n \left( \frac{\lambda_n}{\lambda_2} \right)^k X_n \right)$

Since $|\lambda_1| > |\lambda_0| \ldots > |\lambda_n|$

$C_3 \left( \frac{\lambda_3}{\lambda_2} \right)^k X_3 + \cdots + C_n \left( \frac{\lambda_n}{\lambda_2} \right)^k X_n \to 0 \quad \text{thus} \quad \xi_k \to X_2$