

## HW 6 Sol<sup>4</sup>

3.4 #8)

$$\begin{aligned} x' &= x \\ y' &= y + x^2 \end{aligned} \quad \text{find an integral:}$$

To proceed we use (3.15). ~~at~~

we find an integral of  $(y+x^2)dx - xdy = 0$

This is not exact, so introduce an integrating factor  $m(x)$ .

$$\text{then } \frac{\partial}{\partial y}(m(x)(y+x^2)) = \frac{\partial}{\partial x}(m(x)(-x))$$

$$m(x) = m'(x)(-x) - m(x)$$

$$2m(x) = (-x)m'(x)$$

$$\frac{m'(x)}{m(x)} = \frac{-2}{x}$$

$$\ln m(x) = \ln x^{-2}$$

$$m(x) = \frac{1}{x^2}$$

$$\text{then for } \left(\frac{y}{x^2} + 1\right)dx - \frac{1}{x}dy = 0$$

$$\int -\frac{1}{x} dy = -\frac{y}{x} = Q$$

$$F = Q + H(x)$$

$$P = \frac{\partial}{\partial x} F = \frac{\partial}{\partial x} (Q + H(x)) = \frac{y}{x^2} + H'(x) \quad \text{so } H'(x) = 1$$

$$\text{and } H(x) = x$$

$$\text{so } F = -\frac{y}{x} + x \quad \text{is an integral.}$$

15) The chain rule gives

$$\frac{dy}{dx} \frac{dx}{dt} = \frac{dy}{dx} \frac{dx}{dt} \cdot \frac{dx}{dt} = 1, \text{ then}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \left( \frac{dx}{dt} \right) = \frac{dy}{dx} (1) = \frac{dy}{dx} = f(x, z)$$

Then the system is the same as the eq<sup>n</sup>  $\frac{dy}{dx} = f(x, y)$ .

16) Similar to 15, so we write

$$\frac{dx}{dz} = f(x, y, z)$$

$$\frac{dy}{dz} = g(x, y, z).$$

Using the same trick, the system can be rewritten

$$\frac{dz}{dt} = 1$$

$$\frac{dx}{dt} = \frac{dx}{dz} \frac{dz}{dt} = \frac{dx}{dz} (1) = f(x, y, z)$$

$$\frac{dy}{dz} = \frac{dy}{dz} \left( \frac{dz}{dt} \right) = \frac{dy}{dz} (1) = g(x, y, z)$$

3.5)

#1: a) As the equation has a ~~stable~~ stable ~~node~~ at point  $(d, b/a)$ , we should release 4a predators, which will keep pest population fixed

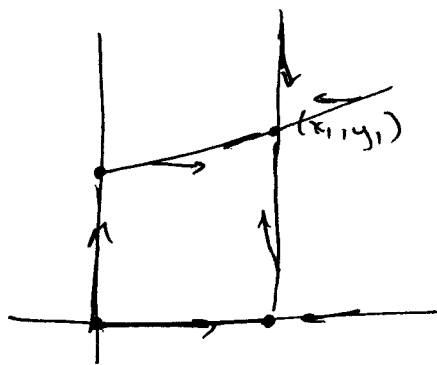
b) This is a bad strategy. Reducing the number of pests will have the effect of moving the population away from the stable point. Regardless of the number of predators released, eventually the pest population will boom.

\*1 c). The ideal strategy is to wait until the pest population approaches  $d$ , then release  $b/a$  predators.

$$\begin{array}{l} 5) \text{ x-nullclines at } x=0 \\ \text{y-nullclines at } y=0 \end{array} \quad \begin{array}{l} K-x+By=0 \\ L+Cx-y=0 \end{array}$$

then there are at most 4 stationary points  
in the case where  $BC < 1$ ,

\* the stationary point at the intersection of the two lines is  
given by  $x_1 = \frac{BL+K}{1-BC}$   $y_1 = \frac{CK+L}{1-BC}$



$$4.1) \text{ ii) } \dot{y} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 3t-5 \\ -6t+4 \end{bmatrix}, \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \end{bmatrix}$$

$$\begin{aligned} \text{then } \begin{bmatrix} 3 \\ -6 \end{bmatrix} &\stackrel{?}{=} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3t-5 \\ -6t+4 \end{bmatrix} + \begin{bmatrix} 9t \\ 0 \end{bmatrix} \\ &\stackrel{?}{=} \begin{bmatrix} 3t-5+2(-6t+4)+9t \\ 2(3t-5)+1(-6t+4)+0 \end{bmatrix} \\ &= \begin{bmatrix} 3 \\ -6 \end{bmatrix}, \text{ so it is a solution.} \end{aligned}$$

$$4) \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}; \quad \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -x \\ y \end{bmatrix}$$

$$\begin{array}{llll} x' = -x & x = C_1 e^{-t} & x(0) = 2 & \Rightarrow C_1 = 2 & x = 2e^{-t} \\ y' = y & y = C_2 e^t & y(0) = 0 & \Rightarrow C_2 = 0 & \end{array}$$

then  $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2e^{-t} \\ 0 \end{bmatrix}$

14) To see that this is a solution, use

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = c_1 \begin{bmatrix} -e^{-t} (2) \\ -e^{-t} (-3) \end{bmatrix} + c_2 \begin{bmatrix} 4e^{4t} \\ 4e^{4t} \end{bmatrix} + \cancel{c_3} \begin{bmatrix} (5)e^{4t} \\ (3)e^{4t} \end{bmatrix}$$

and plug  $\begin{bmatrix} x \\ y \end{bmatrix}$  and  $\begin{bmatrix} x' \\ y' \end{bmatrix}$  into the system.

If  $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$ , then  $\begin{bmatrix} 0 \\ -2 \end{bmatrix} = c_1 \begin{bmatrix} 2 \\ -3 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -5 \\ 3 \end{bmatrix}$

$$\begin{bmatrix} 0 \\ -2 \end{bmatrix} = c_1 \begin{bmatrix} 2 \\ -3 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

and thus  $c_1 = 2, c_2 = 1$ .

$$15) y'' + p(t)y' + q(t)y = r(t)$$

rewrite as system

$$v = y'$$

$$v' = -p(t)v - q(t)y + r(t)$$

then  $\begin{bmatrix} y' \\ v' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ q(t) & -p(t) \end{bmatrix} \begin{bmatrix} y \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ r(t) \end{bmatrix}$

$$4.2) \#3) \begin{bmatrix} 1 & 1 \\ 0 & -4 \end{bmatrix}$$

$$c(s) = s^2 - (-3)s + (-4)$$

$$= s^2 + 3s - 4$$

$$= (s+4)(s-1)$$

$$\text{roots } s_1 = -4 \quad s_2 = 1$$

$$\begin{bmatrix} 1 & 1 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} h \\ k \end{bmatrix} = (-4) \begin{bmatrix} h \\ k \end{bmatrix}$$

$$\begin{bmatrix} h+k \\ -4k \end{bmatrix} = \begin{bmatrix} -4h \\ -4k \end{bmatrix}$$

$$\begin{array}{l} \text{let } h=1 \\ \text{then } k=5 \end{array}$$

so  $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$  is a char. vector for  $s_1 = -4$

$$\begin{bmatrix} 1 & 1 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} h \\ k \end{bmatrix} = (1) \begin{bmatrix} h \\ k \end{bmatrix}$$

$$\begin{bmatrix} h+k \\ -4k \end{bmatrix} = \begin{bmatrix} h \\ k \end{bmatrix}$$

$$\begin{array}{l} \text{let } h=1 \\ \text{then } k=0 \end{array}$$

so  $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$  is a char. vect. for  $s_2 = 1$

$$4) \begin{bmatrix} 3 & 3 \\ -3 & -3 \end{bmatrix}$$

$$c(s) = s^2$$

$s_1 = 0$  is the char value.

$$\text{so } \begin{bmatrix} 3 & 3 \\ -3 & -3 \end{bmatrix} \begin{bmatrix} h \\ k \end{bmatrix} = 0$$

$$\begin{array}{l} \text{let } h=1 \\ \text{then } k=-1. \end{array}$$

$$\underline{\underline{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}}$$