$\boxed{2.1.1}$ (a) $\left[\begin{array}{cc|c} 2 & -3 & 2 \\ 5 & -6 & 8 \end{array}\right]$

subtract $2.5 \times$ row 1 from row 2

$\Rightarrow \left[\begin{array}{cc|c} 2 & -3 & 2 \\ 0 & 1.5 & 3 \end{array}\right]$

$\Rightarrow \begin{cases} 2x - 3y = 2 \\ 1.5y = 3 \end{cases} \Rightarrow \begin{cases} x = 4 \\ y = 2 \end{cases}$

(b) $\left[\begin{array}{cc|c} 1 & 2 & -1 \\ 2 & 3 & 1 \end{array}\right]$

subtract $2 \times$ row 1 from row 2

$\Rightarrow \left[\begin{array}{cc|c} 1 & 2 & -1 \\ 0 & -1 & 3 \end{array}\right]$

$\Rightarrow \begin{cases} x + 2y = -1 \\ -y = 3 \end{cases} \Rightarrow \begin{cases} x = 5 \\ y = -3 \end{cases}$

$\boxed{2.1.3}$ (a) $\begin{cases} 3x - 4y + 5z = 2 & \cdots\cdots (1) \\ 3y - 4z = -1 & \cdots\cdots (2) \\ 5z = 5 & \cdots\cdots (3) \end{cases}$

$\Rightarrow z = 1 \Rightarrow$ Plug $z = 1$ into (2) we have $y = 1$ $\Rightarrow$ Plug $z = 1, y = 1$ into (1) we have $x = 1/3$.

$\boxed{2.2.1}$ (a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

subtract $3 \times$ row 1 from row 2

We have $U = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$

$L = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$

Subtract $2 \times$ row 1 from row 2

We have $U = \begin{bmatrix} 1 & 3 \\ 0 & -4 \end{bmatrix}$

$L = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$\boxed{2.3.2}$ (a) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

step 1: $\|A\|_\infty = 7$

step 2: Find A^{-1}, solve $\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 3 & 4 & 0 \end{array}\right] \Rightarrow$ solution is $\begin{bmatrix} -2 \\ 3/2 \end{bmatrix}$

solve $\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 3 & 4 & 1 \end{array}\right] \Rightarrow$ solution is $\begin{bmatrix} 1 \\ -1/2 \end{bmatrix}$

then $A^{-1} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$

step 3: $\|A^{-1}\|_\infty = 3$

step 4: $\mathrm{cond}(A) = \|A\|_\infty \cdot \|A^{-1}\|_\infty = 21$

(b) $A = \begin{bmatrix} 1 & 2.01 \\ 3 & 6 \end{bmatrix}$

step 1: $\|A\|_\infty = 9$

step 2: Find A^{-1}, solve $\left[\begin{array}{cc|c} 1 & 2.01 & 1 \\ 3 & 6 & 0 \end{array}\right] \Rightarrow$ solution is $\begin{bmatrix} -200 \\ 100 \end{bmatrix}$

solve $\left[\begin{array}{cc|c} 1 & 2.01 & 0 \\ 3 & 6 & 1 \end{array}\right] \Rightarrow$ solution is $\begin{bmatrix} 67 \\ -\frac{100}{3} \end{bmatrix}$

$\Rightarrow A^{-1} = \begin{bmatrix} -200 & 67 \\ 100 & -100/3 \end{bmatrix}$

step 3. $\|A^{-1}\|_\infty = 267$

step 4. $\text{Cond}(A) = \|A\|_\infty \|A^{-1}\|_\infty = 2403$

$\boxed{2.3.8}$ (a) Matrix is $A = \begin{bmatrix} 1 & 1 \\ 1+\delta & 1 \end{bmatrix}$, $\delta > 0$.

step 1: $\|A\|_\infty = 2+\delta$

step 2: Find A^{-1}, solve $\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 1+\delta & 1 & 0 \end{array}\right] \Rightarrow$ solu is $\begin{bmatrix} -1/\delta \\ \frac{1+\delta}{\delta} \end{bmatrix}$

solve $\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 1+\delta & 1 & 1 \end{array}\right] \Rightarrow$ solu is $\begin{bmatrix} 1/\delta \\ -1/\delta \end{bmatrix}$

$\Rightarrow A^{-1} = \begin{bmatrix} -\frac{1}{\delta} & 1/\delta \\ \frac{1+\delta}{\delta} & -1/\delta \end{bmatrix}$

step 3: $\|A^{-1}\|_\infty = \dfrac{2+\delta}{\delta}$

step 4: $\text{Cond}(A) = \|A\|_\infty \cdot \|A^{-1}\|_\infty = \dfrac{(2+\delta)^2}{\delta}$

(b) step 1: Find relative forward error.

The exact soln $X = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\Rightarrow \quad \|X - X_a\|_\infty = 2+\delta$

$\|X\|_\infty = 1$

$\Rightarrow$ relative forward error is $2+\delta$.

step 2: Find relative backward error.

$r = b - AX_a = \begin{bmatrix} -\delta \\ \delta \end{bmatrix}$, and $\|b\|_\infty = 2+\delta$

$\Rightarrow$ relative backward error is $\dfrac{\|r\|_\infty}{\|b\|_\infty} = \dfrac{\delta}{2+\delta}$

step 3: The error magnification factor

$$= \frac{RFE}{RBE} = \frac{(2+\delta)^2}{\delta}$$

$\boxed{2.4.3}$ (b).

step 1. Eliminate ~~the~~ column 1.

Exchange Row 1 and 2., $\Rightarrow P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 6 & 3 & 4 \\ 3 & 1 & 2 \\ 3 & 1 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 & 3 & 4 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{1}{2} & 3 \end{bmatrix}$$

step 2. Eliminate column 2.

No Row exchange needed

$$\begin{bmatrix} 6 & 3 & 4 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{1}{2} & 3 \end{bmatrix} = \begin{bmatrix} 6 & 3 & 4 \\ \frac{1}{2} & -0.5 & 0 \\ \frac{1}{2} & 1 & 3 \end{bmatrix}.$$

Then, we have

$$P=\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix},\quad L=\begin{bmatrix}1&&\\\frac{1}{2}&1&\\\frac{1}{2}&1&1\end{bmatrix}$$

$$U=\begin{bmatrix}6&3&4\\&-0.5&0\\&&3\end{bmatrix}$$

step 3. Solve $Lc=Pb$

By back substitution $\begin{bmatrix}1&&\\\frac{1}{2}&1&\\\frac{1}{2}&1&1\end{bmatrix}\cdot\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}=\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}\begin{bmatrix}0\\1\\3\end{bmatrix}$

$$\Rightarrow c=\begin{bmatrix}1\\-\frac{1}{2}\\3\end{bmatrix}$$

step 4. Solve $Ux=c$

By back substitution $\begin{bmatrix}6&3&4\\&-0.5&0\\&&3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}1\\-\frac{1}{2}\\3\end{bmatrix}$

$$\Rightarrow x=\begin{bmatrix}-1\\1\\1\end{bmatrix}.$$

$\boxed{2.5.1}$ (c) Jacobi iteration

$$x_{k+1}=D^{-1}(b-(L+U)x_k) \quad \cdots\cdots (1)$$

where $D=\begin{bmatrix}3&&\\&3&\\&&3\end{bmatrix}$, $L=\begin{bmatrix}0&&\\1&0&\\1&1&0\end{bmatrix}$, $U=\begin{bmatrix}0&1&1\\&0&1\\&&0\end{bmatrix}$

$$b=\begin{bmatrix}6\\3\\5\end{bmatrix},\quad x_0=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$\Rightarrow$ Plug everything into (1)

One has $x_1=\begin{bmatrix}2\\1\\5/3\end{bmatrix}$, $\quad x_2=\begin{bmatrix}\frac{10}{9}\\-\frac{2}{9}\\2/3\end{bmatrix}$.

Gauss - Seidal iteration

$$(D+L)X_{k+1} = b - UX_k \quad \cdots\cdots (2)$$

Plug D, L, U, b into (2)

We have $X_1 = \begin{bmatrix} 2 \\ 1/3 \\ 8/9 \end{bmatrix}$, $X_2 = \begin{bmatrix} 1.5926 \\ 0.1728 \\ 0.8889 \end{bmatrix}$.

$\boxed{2.6.1}$ (a) $\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} = A$

Obviously, $A^T = A$, A is symmetric.

And, $X^TAX = X_1^2 + 3X_2^2 > 0$, $\forall X \neq 0$

thus, A is symmetric pos-def.

(c). $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Obviously, $A^T = A$, A is symmetric.

And, $X^TAX = X_1^2 + 2X_2^2 + 3X_3^2 > 0$, $\forall X \neq 0$,

thus A is sym pos-def.

2.6.13

(a). $A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Let $X_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow d_0 = r_0 = b - AX_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

step 1: $\alpha_0 = \dfrac{r_0^T r_0}{d_0^T A d_0} = \dfrac{1}{5}$

$x_1 = X_0 + \alpha_0 d_0 = \begin{bmatrix} \frac{1}{5} \\ \frac{1}{5} \end{bmatrix}$

$r_1 = r_0 - \alpha_0 A d_0 = \begin{bmatrix} 2/5 \\ -2/5 \end{bmatrix}$

$\beta_0 = \dfrac{r_1^T r_1}{r_0^T r_0} = \dfrac{4}{25}$

$d_1 = r_1 + \beta_0 d_0 = \begin{bmatrix} \frac{14}{25} \\ -\frac{6}{25} \end{bmatrix}$

step 2: $\alpha_1 = \dfrac{r_1^T r_1}{d_1^T A d_1} = 5$

$X_2 = x_1 + \alpha_1 d_1 = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

$r_2 = r_1 - \alpha_1 d_1 = 0$ stop.

Solu is $x = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$.

$\boxed{2.7.1}$ (a) $F(u,v)=(u^3, uv^3)$

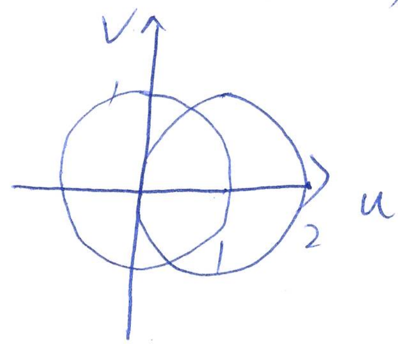
$$J=\begin{bmatrix} \frac{\partial f_1}{\partial u} & \frac{\partial f_1}{\partial v} \\ \frac{\partial f_2}{\partial u} & \frac{\partial f_2}{\partial v} \end{bmatrix} = \begin{bmatrix} 3u^2 & 0 \\ v^3 & 3uv^2 \end{bmatrix}$$

(d) $F(u,v,w)=(u^2+v-w^2,\ \sin uvw,\ uvw^4)$

$$J=\begin{bmatrix} \frac{\partial f_1}{\partial u} & \frac{\partial f_1}{\partial v} & \frac{\partial f_1}{\partial w} \\ \frac{\partial f_2}{\partial u} & \frac{\partial f_2}{\partial v} & \frac{\partial f_2}{\partial w} \\ \frac{\partial f_3}{\partial u} & \frac{\partial f_3}{\partial v} & \frac{\partial f_3}{\partial w} \end{bmatrix} = \begin{bmatrix} 2u & 1 & -2w \\ vw\cos uvw & uw\cos uvw & uv\cos uvw \\ vw^4 & uw^4 & 4uvw^3 \end{bmatrix}$$

$\boxed{2.7.3}$ (a) $\begin{cases} u^2+v^2=1 \cdots (1) \\ (u-1)^2+v^2=1 \cdots (2) \end{cases}$

Graph:

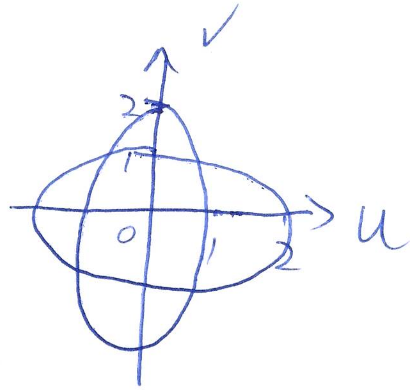


From (2), we have $v^2=1-(u-1)^2$, plug it into (1),

One has $u^2-(u-1)^2=0 \Rightarrow u-1=-u \Rightarrow u=\frac{1}{2}$

$\Rightarrow v=\pm\frac{\sqrt{3}}{2} \Rightarrow$ Soln are $\begin{cases} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{cases}$, $\begin{cases} \frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{cases}$

(b). $\begin{cases} u^2+4v^2=4 \cdots (1) \\ 4u^2+v^2=4 \cdots (2) \end{cases}$ Graph.



From (2), one has $v^2=4-4u^2$
substitute it into (1).
$u^2+16-16u^2=4$

$\Rightarrow u=\pm\frac{2}{\sqrt5}, \Rightarrow v=\pm\frac{2}{\sqrt5}$

$\Rightarrow$ Solu are $\begin{cases} \frac{2}{\sqrt5} \\ \frac{2}{\sqrt5} \end{cases}$, $\begin{cases} -\frac{2}{\sqrt5} \\ \frac{2}{\sqrt5} \end{cases}$, $\begin{cases} \frac{2}{\sqrt5} \\ -\frac{2}{\sqrt5} \end{cases}$, $\begin{cases} -\frac{2}{\sqrt5} \\ -\frac{2}{\sqrt5} \end{cases}$

$\boxed{2.7.4}$ (a). $J=\begin{bmatrix} \frac{\partial f_1}{\partial u} & \frac{\partial f_1}{\partial v} \\ \frac{\partial f_2}{\partial u} & \frac{\partial f_2}{\partial v} \end{bmatrix} = \begin{bmatrix} 2u & 2v \\ 2(u-1) & 2v \end{bmatrix}$, $F=\begin{cases} u^2+v^2-1 \\ (u-1)^2+v^2-1 \end{cases}$

step 1. $J(X_0)h=-F(X_0)$

$\Rightarrow \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} h = -\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\Rightarrow h=[-\frac12 \ \ 0]^T \Rightarrow X_1=X_0+h=[\frac12 \ \ 1]^T$.

step 2. $J(X_1)h=-F(X_1)$

$\Rightarrow \begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix} h = -\begin{bmatrix} \frac14 \\ \frac14 \end{bmatrix}$

$\Rightarrow h=[0, -\frac18]^T \Rightarrow X_2=X_1+h=[\frac12, \frac78]^T$

(b). $J=\begin{bmatrix} 2u & 8v \\ 8u & 2v \end{bmatrix}$, $F=\begin{cases} u^2+4v^2-4 \\ 4u^2+v^2-4 \end{cases}$

step1. $J(X_0)h=-F(X_0)$

$\Rightarrow \begin{bmatrix} 2 & 8 \\ 8 & 2 \end{bmatrix} h = -\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\Rightarrow h=[-\frac{1}{10}, -\frac{1}{10}]^T \Rightarrow X_1=X_0+h = [\frac{9}{10}, \frac{9}{10}]^T$.

step2. $J(X_1)h=-F(X_1)$

$\Rightarrow \begin{bmatrix} \frac95 & \frac{36}{5} \\ \frac{36}{5} & \frac95 \end{bmatrix} h = -\begin{bmatrix} \frac{1}{20} \\ \frac{1}{20} \end{bmatrix}$

$\Rightarrow h=[-\frac{1}{180}, -\frac{1}{180}]^T$

$\Rightarrow X_2=X_1+h=[\frac{161}{180}, \frac{161}{180}]^T$