HW3 MATH170B.

#3.2: (i) $g(\bar{x}) \neq 0$, fails 1st order condition,

thus $\bar{x}$ is not a local optimizer.

(ii) $g(\bar{x}) = 0$, 1st order condition ✓

$H(\bar{x}) = \begin{pmatrix} 3 & 2 \\ 2 & 0 \end{pmatrix}$, solve for $\det(H - \lambda I) = 0$

$\Rightarrow \lambda_1 = 4, \lambda_2 = -1$

$\Rightarrow H(\bar{x})$ is not pos-def nor neg-def

Fails 2nd order condition.

Thus $\bar{x}$ is not a local optimizer.

(iii) $g(\bar{x}) = 0$, 1st order condition ✓

$H(\bar{x}) = \begin{pmatrix} 4 & -1 \\ -1 & 4 \end{pmatrix}$, $\Rightarrow \lambda_1 = 5, \lambda_2 = 3$

$\Rightarrow H(\bar{x})$ is pos-def

$\Rightarrow$ 2nd order condition ✓

$\bar{x}$ is a local minimazer.

(iv) $g(\bar{x}) = 0$, 1st order condition ✓

$H(\bar{x}) = \begin{pmatrix} -2 & 0 \\ 0 & -3 \end{pmatrix} \Rightarrow \lambda_1 = -2, \lambda_2 = -3$

$\Rightarrow H(\bar{x})$ is ~~not positive de~~ neg-def

$\Rightarrow$ 2nd order condition ✓

$\bar{x}$ is a local maximazer.

(v). $g(\bar{x}) = 0$, 1st order condition ✓

$$H(\bar{x}) = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \Rightarrow \lambda_1 = 0,\ \lambda_2 = 2,\ \lambda_3 = 3$$

$\Rightarrow H(\bar{x})$ is not pos-def nor neg-def

$\Rightarrow$ 2nd order condition fails. ↳ semi-pos-def

~~not a local optimizer.~~

$\bar{x}$ may be local minimizer, information is not enough

#3.3.

(a). $\nabla q(x) = c^T + x^T H$ or $c + Hx$

(b). $q(x_0 + p) = q(x_0) + p^T \nabla q(x_0) + \frac{1}{2} p^T H(x_0) p$

$= q(x_0) + p^T(c + Hx) + \frac{1}{2} p^T H p.$

(c). Proof: We have

$$\begin{cases} \dfrac{d\, q(x_0 + \alpha p)}{d\alpha} = c^T p + (x_0 + \alpha p)^T H p & \cdots \text{①} \\ \\ \dfrac{d^2 q(x_0 + \alpha p)}{d\alpha^2} = p^T H p & \text{②} \end{cases}$$

Since ② > 0 for any $p \neq 0$ (H is pos-def), thus 2nd order condition always hold.

Then if ① $= 0$ has a root, then this root will be a minimizer.

Now solve $① = 0 \Rightarrow \alpha^* = \dfrac{-c^T p - x_0^T H p}{p^T H p}$

Last step is to show α^* is positive

Since $\nabla q(x_0)^T p < 0 \Rightarrow -c^T p - x_0^T H p > 0$

$\Rightarrow \alpha^* > 0$.

$\square$.

#3.5.

Proof: (a) f is convex.

Since x^* is stationary point, i.e. $f'(x^*) = 0$

Then $\forall y$, $f(y) \geq f(x^*) + f'(x^*)(y - x^*) = f(x^*)$.

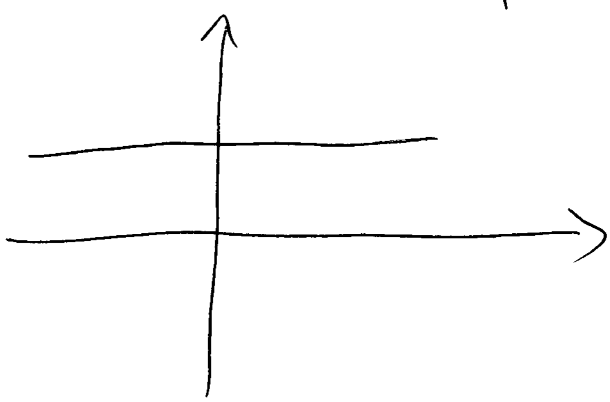
thus x^* is a global minimizer.

(b) f is strictly convex.

Similarly, $\forall y$ $f(y) > f(x^*) + f'(x^*)(y - x^*) = f(x^*)$

thus x^* is unique global minimizer.

(c). Consider constant function $f(x) = 1$



it is convex, has infinite many global min.