General Outline and Reading Guide for the Lectures

The following is a chronological list of the topics we will cover in the next ten weeks. We will spend roughly a week on each topic. Below each topic is a list of the corresponding chapters in the textbook covering the material. While we will cover the important theoretical concepts precisely and rigorously, we will devote an equal amount of time and energy to the art of implementation, and to the application of the methods to real problems in biology, chemistry, engineering, mathematics, and physics.

1. Introduction to partial differential equations (PDE) and numerical solution methods
   (EEHJ Chapters: 1, 2, and supplementary material)

2. Crash course in linear analysis and some tools from nonlinear analysis
   (EEHJ Chapters: 3, 4, 13, 21, and supplementary material)

3. Polynomials, the Petrov-Galerkin method, and approximation theory
   (EEHJ Chapters: 5, 6, 14, and supplementary material)

4. Elliptic PDE: classical and weak solutions, calculus of variations
   (EEHJ Chapters: 8, 11, 13, 15, and supplementary material)

5. Finite element methods: Petrov-Galerkin formulation and computer implementation
   (EEHJ Chapters: 12, 6, 7, and supplementary material)

6. Finite element approximation theory
   (EEHJ Chapters: 8, 14, 15, 21, and supplementary material)

7. Geometry of simplices: mesh generation, simplex subdivision, and mesh conformity
   (EEHJ Chapters: 14, and supplementary material)

8. Adaptivity through simplex subdivision and *a posteriori* error estimation
   (EEHJ Chapters: 8, 14, 15, and supplementary material)

A Rough Outline of Math 105ABC or Equivalent

A basic numerical analysis course (Math 105ABC or equivalent) is not a prerequisite for this course; however, in the future we will try to enforce this. Since we are not enforcing it this year, we will present some of this basic material as we need it at various points. For your general information, the material typically covered in a basic numerical analysis course such as Math 105ABC is listed below.

1. Math 105A: Interpolation and approximation of functions and data

   (a) Iterative methods for nonlinear equations in one variable

   (b) Polynomial interpolation

   (c) Quadrature and numerical differentiation

   (d) Classical (polynomial) approximation theory

2. Math 105B: Solving linear and nonlinear algebraic systems of equations

   (a) Direct methods for linear systems (dense and sparse Gaussian elimination)

(b) Iterative methods for linear systems (stationary methods and the CG method)

(c) Methods for linear eigenvalue problems

(d) Iterative methods for nonlinear equations in several variables

3. Math 105C: Numerical solution of ordinary and partial differential equations

(a) Solving boundary- and initial-value problems in ordinary differential equations

(b) Methods for elliptic partial differential equations

(c) Methods for parabolic partial differential equations

(d) Methods for hyperbolic partial differential equations

Homework, Exams, and Grading Policies

- Lecture attendance is strongly encouraged

- Theoretical homeworks will be assigned about every two weeks, based on the lectures.

- Computer homeworks will be assigned about every two weeks.

- There will be a final exam.

- Grading will be: 40% homework, 30% computer project, 30% final.

- Homeworks will be due at the beginning of the lecture on due date, and there will be 10% off for each day the homework is late.

The computer homeworks represent pieces of a class project. The computer homeworks may be completed in any of C, C++, FORTRAN, or MATLAB (you will want to use the same language for all of the projects, since they will need to fit together). We will have access to a computer lab running MATLAB throughout the quarter.

NOTE: Appearing in the lab at the reserved time for this class is not required; you may complete your lab assignments on any platform you have access to, whenever you see fit to do it (as long as you turn it in on time). However, from time to time I will give some mini-lectures in the lab during the lab hour, to clear up various implementation issues.

Electronic Resources for the Class

We will try to communicate via email; my email address is `mholst@uci.edu`. The class web page is currently:

`http://www.math.uci.edu/~mholst/teaching/uci/math292a.html`

Homeworks will be posted to this page, usually in postscript form. Announcements and other things will be posted periodically, so you should monitor it regularly.

OAC provides an email list alias and archive service; if I can sort out how to use this, we will give it a try. They also provide web page space; I may move the above web page to the OAC site (I will let you know when and if this happens).

Books and Reference Material

The main text is [16], with supplementary material coming from [6, 21, 26], as well as from a few other sources listed below. The subject of this course can be seen as lying at the intersection of the following three general areas:

Differential Equations: The branch of mathematics concerned with the study of ordinary and partial differential equations which arise in all areas of mathematics and science. Analysis of differential equations, especially partial differential equations, often involves the tools of **functional analysis**. Most differential equations can be solved analytically only in very special situations, and as a result the algorithms developed in **numerical analysis** must often be employed in mathematics, science, and engineering.

Numerical Analysis: The branch of mathematics concerned with the study of computation and of its accuracy, stability, and often its implementation on a computer. One central concern is the determination of appropriate numerical models for applied problems. Another is the construction and analysis of robust and efficient algorithms for various mathematical problems such as those of numerical integration and differentiation, and combinatorial and constrained optimization problems. One area of numerical analysis which finds use in many other areas of science and engineering is the construction and analysis of methods for the numerical solution of ordinary and partial **differential equations** of all kinds. The analysis of such differential equations as well as the analysis of the appropriate numerical algorithms often requires tools from **functional analysis**. (Adapted with much liberty from the Harper Collins Mathematics Dictionary.)

Functional Analysis: The branch of mathematics concerned with the modern abstract study of linear and non-linear functions in terms of the underlying linear (topological, normed, Banach, Hilbert, ...) spaces on which the functions are defined and the duals of those spaces. This perspective, growing out of the study of linear operators and functionals, aims at producing a unifying corpus of results and techniques for linear spaces and linear operators. This is applicable to the study of such diverse areas of mathematics as algebra, real analysis, **numerical analysis**, calculus of variations, and **differential equations**, through the application of general theorems such as the Hahn-Banach Theorem, the Uniform Boundedness Principle, the Open Mapping Theorem, and the Riesz Representation Theorem. (Twisted somewhat from the Harper Collins Mathematics Dictionary.)

Below is a list of what I feel are some of the more important reference books for this "intersection" area of elliptic equations, numerical analysis, and functional analysis. The list is a little excessive but you can use it as a guide. For particular topics, I'll supplement material from a few of these books. I have all of books on the list below in my office (with a few exceptions) so feel free to come by and browse my library.

- General numerical analysis: [11, 12, 25, 43, 52]

- General numerical treatment of elliptic equations: [6, 16, 21, 23, 46]

- Finite element theory: [3, 4, 6, 7, 10, 16, 21, 42, 45]

- Finite element implementation: [3, 5, 6, 16, 26]

- Iterative methods for linear problems: [22, 53, 54, 56]

- Iterative methods for nonlinear problems: [15, 28, 38, 44]

- Multigrid and domain decomposition methods: [9, 20, 49]

- Linear elliptic equations: [2, 18, 21, 37, 40, 47, 50, 51]

- Nonlinear elliptic equations: [17, 18, 41]

- Real analysis: [27, 31, 39, 48]

- Functional analysis: [8, 14, 28, 30, 36, 55]

- Approximation theory: [13, 45]

- Operator and matrix theory [19, 22, 24, 29, 32, 53]

- Sobolev and Besov spaces: [1, 33, 34, 35]

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