# MATRIX THEORY

## Linear operators on finite-dimensional spaces

**(Lectures for Fall 1996 AMa 104)**

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# Contents of the Lectures

NOTES: Following the comments below, we will study the material in Part 2 for most of the course, looking back at material from Part 1 when it is appropriate. We will however start out with an introduction to the material in Part 1 for the first few lectures, before diving in Part 2. Part 1 can be seen as a collection of the fundamental, general, and extremely important ideas which the material in Part 2 rests on, but which are often lost along the way.

- **Part 1: Vector Spaces and Linear Transformations**
  1. Sets, mappings, and linearity
  2. Fields, orderings, metrics, and metric spaces
  3. Vector (or Linear) spaces, norms, and inner-products
  4. Normed and inner-product spaces and their geometries
  5. Linear transformations (or operators) on vector spaces
  6. Continuity, boundedness, and norms of linear operators
  7. Adjoints, linear functionals, and bilinear forms
  8. Examples of linear operators on vector spaces
- **Part 2: Bases, Matrices, Operators, and Applications**
  1. Linear independence, bases, and matrices
  2. Linear systems of equations and gaussian elimination
  3. Orthogonality, projections, and least squares
  4. Properties and applications of determinants
  5. Eigenvalues, eigenvectors, and the Jordon form
  6. Positive definite matrices and some applications
  7. Norms, conditions numbers, and matrix computations
  8. Some applications of matrix theory

# AMa 104
# Matrix Theory

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| Term: | Fall 1996 |
| Place & Time: | Firestone 102, 1:00pm-2:00pm, M-W-TH |
| Instructor: | Michael Holst (holst@ama.caltech.edu), 313 Firestone, Caltech, x4549 |
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In this course, we will cover the theory of linear (or vector) spaces, linear operators on vector spaces, the theory of matrices, and applications of matrix theory. Throughout the lectures, we will stress the central role which an abstract linear operator plays in the theory of linear spaces and in applications in which matrices arise. Although this course is entitled "Matrix Theory", it is often somewhat surprising to students to learn that linear algebra is not mainly about "matrices", and in fact an entire linear algebra course can be done with almost no reference to a matrix!

A linear algebra course taught in such a "coordinate-free" approach has the advantage that it presents the fundamentally important concepts of linear operators, linear spaces, and inner-products and norms, without presenting the "matrix" as the star of the show. Moreover, many of these concepts carry over to the infinite-dimensional setting (alas, matrices do not), so that a thorough understanding of operators on finite-dimensional vector spaces can provide a solid foundation for learning the theory of differential and integral operators, for example. This view is held by Halmos [4], and we will start out with his book for the first few lectures of the course as an introduction to linear transformations on vector spaces.

Of course, we are often faced in applications with the practical need to do things like solve (possibly large and sparse) linear systems of equations, determine (at least some of) the eigenvalues and/or eigenvectors of a particular matrix (which again may be large and sparse), or analyze a differential equation by investigating the properties of the associated matrices. Therefore, we will spend most of the course with the book of Strang [9] learning about matrices and applications of matrix theory. However, we will often reach back to the book of Halmos when it seems important to stress the generality of the ideas.

Numerical algorithms for matrix equations will not be covered in this course (although we will make some remarks about these algorithms from time to time during the course); such algorithms will be covered in great detail next quarter in AMa105b.

## Requirements, Homework, Exams, etc

- The main requirement is to learn the material.
- Attending class is encouraged (discussing is the best way to learn; also I like giving lectures).
- Homeworks will be assigned about every two weeks (for a total of 4 of equal weight).
- If one of the homeworks contains a machine problem, it can be done in MATLAB.
- There will be midterm and final exams, having the weight of two and three homeworks, respectively.
- Discussion is allowed and encouraged on the homeworks.
- Your grade will be based on the four homeworks and on the midterm and final exams.

## Books and Reference Material

The book of Strang [9] covers most of matrix-oriented material in the course, as well as applications of matrix theory. The book of Halmos [4] presents some of the same material, but with a "coordinate-free" approach; the underlying linear operators are analyzed rather than matrices resulting from an arbitrary basis choice. Here is a list of my favorite reference books on linear spaces, linear operator theory, matrices, applications, and numerical methods for matrix equations: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10].

## From the Prefaces to [4]

"My purpose in this book is to treat linear transformations on finite-dimensional vector spaces by the methods of more general theories. The idea is to emphasize the simple geometric notions common to many parts of mathematics and its applications, and to do so in a language that gives away the trade secrets and tells the student what is in the back of the minds of people proving theorems about integral equations and Hilbert spaces...the algebraic, coordinate-free methods do not lose power and elegance by specialization to a finite number of dimensions, and they are, in my belief, as elementary as the classical coordinatized [matrix-oriented] treatment..."

Translation: Rather than present linear algebra as a collection of "tricks" for manipulating matrices, Halmos tries to present it as a uniform framework for understanding linear operators on vector spaces, equally applicable to matrices and vectors in $\mathbb{R}^n$ as well as to linear integral and differential operators on vector spaces of functions.

## References

[1] J. Franklin. *Matrix Theory.* Prentice-Hall, Englewood Cliffs, NJ, 1968.

[2] G. H. Golub and C. F. Van Loan. *Matrix Computations.* The Johns Hopkins University Press, Baltimore, MD, second edition, 1989.

[3] W. Hackbusch. *Iterative Solution of Large Sparse Systems of Equations.* Springer-Verlag, Berlin, Germany, 1994.

[4] P. R. Halmos. *Finite-Dimensional Vector Spaces.* Springer-Verlag, Berlin, Germany, 1958.

[5] E. Isaacson and H. B. Keller. *Analysis of Numerical Methods.* John Wiley & Sons, Inc., New York, NY, 1966.

[6] L. V. Kantorovich and G. P. Akilov. *Functional Analysis.* Pergamon Press, New York, NY, 1982.

[7] T. Kato. *Perturbation Theory for Linear Operators.* Springer-Verlag, Berlin, Germany, 1980.

[8] E. Kreyszig. *Introductory Functional Analysis with Applications.* John Wiley & Sons, Inc., New York, NY, 1990.

[9] G. Strang. *Linear Algebra and its Applications.* Harcourt Brace Jovanovich, Inc., Orlando, FL, 1988.

[10] R. S. Varga. *Matrix Iterative Analysis.* Prentice-Hall, Englewood Cliffs, NJ, 1962.