

BOUNDARY VALUE PROBLEMS



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Theory and Applications

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To Lainie and Alissa.
-I.S.

For Mai,
Mason, and Makenna.
-M.H.



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