Introduction to Binary Black Hole Evolutions

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Generalized Harmonic Einstein Equations

- Specify the spacetime coordinates $x^a$ using a gauge source function $H^a(x, \psi)$:

$$H^a(x, \psi) = \nabla^c \nabla_c x^a = \psi^{bc} \partial_b \partial_c x^a + \ldots,$$

where $\psi_{ab}$ is the spacetime metric.

- The Einstein equations become manifestly hyperbolic:

$$\psi^{cd} \partial_c \partial_d \psi_{ab} = \nabla_a H_b + \nabla_b H_a + F_{ab}(\psi, \partial \psi).$$

- Adding certain multiples of the constraints to these equations gives them excellent constraint damping properties.
Moving Black Holes

Problem: Causality issues when black holes move through a computational domain:

Solution: Choose coordinates that smoothly track the location of the black hole.

Use co-rotating coordinates for widely separated binaries.
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- Use co-rotating coordinates for widely separated binaries.
Evolving Black Holes in Rotating Frames

- Evolutions of Schwarzschild in rotating coordinates are unstable.

\[ \Omega = \frac{0.2}{M} \]
\[ \Omega = \frac{0.02}{M} \]
\[ \Omega = \frac{0.002}{M} \]
\[ \Omega = \frac{0.0002}{M} \]

\[ \left\| C \right\| \]

Problem caused by asymptotic behavior of metric in rotating coordinates:
\[ \psi_{tt} \sim \rho^2 \Omega^2, \]
\[ \psi_{ti} \sim \rho \Omega, \]
\[ \psi_{ij} \sim 1. \]
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- Problem caused by asymptotic behavior of metric in rotating coordinates: $\psi_{tt} \sim \rho^2\Omega^2$, $\psi_{ti} \sim \rho\Omega$, $\psi_{ij} \sim 1$. 
Dual-Coordinate-Frame Evolution Method

- Coordinates serve two different purposes:
  - The coordinate basis defines the components of tensor fields:
    \[ \psi = \psi_{ab} \, dx^a \otimes dx^b. \]
  - Field components are determined as functions of the coordinates:
    \[ \psi_{ab} = \psi_{ab}(x^c). \]

- Use different coordinates for different purposes:
  - Use asymptotically inertial coordinates to define field components.
  - Use co-moving coordinates to evaluate these functions by solving Einstein's equation.
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Testing Dual-Frame Evolutions

- Single-frame evolutions of Schwarzschild in rotating coordinates are unstable, while dual-frame evolutions are stable:

![Graph showing Dual Frame Evolution and Single Frame Evolution with different frequencies and NR values](image-url)
Horizon Tracking Coordinates

Choose co-moving coordinates that track the black holes:

\[
\begin{pmatrix}
  x \\
y \\
z
\end{pmatrix} = e^{a(t)} \begin{pmatrix}
  \cos \varphi(t) & -\sin \varphi(t) & 0 \\
  \sin \varphi(t) & \cos \varphi(t) & 0 \\
  0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
  \bar{x} \\
  \bar{y} \\
  \bar{z}
\end{pmatrix}.
\]

Measure the comoving positions of the holes:

\[
Q_x(t) = \frac{x_c(t) - x_c(0)}{x_c(0)},
\]

\[
Q_y(t) = \frac{y_c(t)}{x_c(t)}.
\]

Choose the map parameters \(a(t)\) and \(\varphi(t)\) to keep \(Q_x(t)\) and \(Q_y(t)\) small.
Horizon Tracking Coordinates II

- In the time interval $t_i < t < t_{i+1}$ choose the map parameters:

$$\varphi(t) = \varphi_i + (t - t_i) \frac{d\varphi_i}{dt} + \frac{(t - t_i)^2}{2} \frac{d^2 \varphi_i}{dt^2}$$

$$+ \frac{(t - t_i)^3}{2} \left( \lambda \frac{d^2 Q^y_i}{dt^2} + \lambda^2 \frac{d Q^y_i}{dt} + \lambda^3 \frac{Q^y_i}{3} \right).$$

- This choice drives $Q^y(t)$ exponentially toward zero on the timescale $1/\lambda$. 

This works! We are now able to evolve binary black holes using horizon tracking coordinates until just before merger.
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- **This works!** We are now able to evolve binary black holes using horizon tracking coordinates until just before merger.