Model Waveform Accuracy Standards for Gravitational Wave Data Analysis

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Outline

- Derive model waveform accuracy requirements for ideal detectors:
  - Standards for measurement.
  - Standards for detection.

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- What this talk will not cover
    - How to measure NR waveform errors.
    - How well do current NR waveforms satisfy these standards.
    - ...

Accuracy Standards for Measurement

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- The variance for measuring the parameter λ is given by

$$\frac{1}{\sigma_\lambda^2} = \left\langle \frac{\partial h}{\partial \lambda} \middle| \frac{\partial h}{\partial \lambda} \right\rangle = \langle \delta h | \delta h \rangle,$$

where the noise weighted inner product is defined by

$$\langle h_e | h_m \rangle = 2 \int_0^\infty \frac{h_e^*(f) h_m(f) + h_e(f) h_m^*(f)}{S_n(f)} df.$$

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- Two waveforms are indistinguishable iff the variance σ_λ^2 is larger than the parameter distance between the waveforms: $(\Delta\lambda)^2 = 1 < \sigma_\lambda^2 = 1/\langle \delta h | \delta h \rangle$, that is iff $1 > \langle \delta h | \delta h \rangle$.

Accuracy Standards for Detection

- The signal-to-noise ratio ρ_m for detecting a signal h_e using a filter constructed from a model waveform h_m is

$$\rho_m = \langle h_e | \hat{h}_m \rangle = \frac{\langle h_e | h_m \rangle}{\langle h_m | h_m \rangle^{1/2}}.$$

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$$\epsilon = \frac{\langle \delta h_\perp | \delta h_\perp \rangle}{2\langle h_e | h_e \rangle}, \quad \text{where} \quad \delta h_\perp = \delta h - h_e \langle h_e | \delta h \rangle / \rho^2.$$

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- For detection, model waveform accuracy must satisfy the requirement $\langle \delta h_\perp | \delta h_\perp \rangle < 2\epsilon_{\max}\rho^2$.

Sufficient Conditions for Waveform Accuracy

- The optimal accuracy standards $\langle \delta h | \delta h \rangle < 1$ and $\langle \delta h_\perp | \delta h_\perp \rangle < 2\epsilon_{\rm max}\, \rho^2$ depend in the details of the waveform model (e.g. the total mass and location of the source) as well as the details of the detector noise spectrum $S_n(f)$.
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- Construct slightly stronger *sufficient* conditions that are easier for the NR community to use.
- One simplification can be made by noting that

$$\langle\delta h_\perp|\delta h_\perp\rangle \le \langle\delta h|\delta h\rangle,$$

So a sufficient condition for detection is:

$$\langle\delta h|\delta h\rangle < 2\epsilon_{\rm max}\,\rho^2.$$

Sufficient Conditions for Waveform Accuracy II

- Define the model waveform (logarithmic) amplitude $\delta\chi$ and phase $\delta\Phi$ errors: $\delta h = h_e(\delta\chi + i\delta\Phi)$.

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- The basic accuracy requirements can be written as

$$\frac{\langle \delta h | \delta h \rangle}{\rho^2} = \overline{\delta\chi}^2 + \overline{\delta\Phi}^2 < \begin{cases} 1/\rho^2 & \text{measurement,} \\ 2\epsilon_{\max} & \text{detection,} \end{cases}$$

where the signal-weighted average errors are defined as

$$\overline{\delta\chi}^2 = \int_0^\infty \delta\chi^2 \frac{4|h_e|^2}{\rho^2 S_n} df, \quad \text{and} \quad \overline{\delta\Phi}^2 = \int_0^\infty \delta\Phi^2 \frac{4|h_e|^2}{\rho^2 S_n} df.$$

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- These averages satisfy $\overline{\delta\Phi} \le \max|\delta\Phi|$, etc., so a set of sufficient accuracy requirements are

$$\left(\max|\delta\chi|\right)^2 + \left(\max|\delta\Phi|\right)^2 < \begin{cases} 1/\rho^2 & \text{measurement,} \\ 2\epsilon_{\max} & \text{detection.} \end{cases}$$

Sufficient Conditions for Waveform Accuracy III

- We can derive another sufficient waveform accuracy requirement by noting that:

$$\langle \delta h | \delta h \rangle = 4 \int_0^\infty \frac{|\delta h|^2}{S_n(f)} df \le \frac{2 ||\delta h(f)||^2}{\min S_n(f)},$$

where $||\delta h(f)||^2 = 2 \int_0^\infty |\delta h|^2 df$ is the L^2 norm of $\delta h(f)$.

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- We can therefore convert the basic accuracy requirements into the following sufficient conditions:
$$\frac{||\delta h(f)||^2}{||h_e(f)||^2} < \begin{cases} C^2/\rho^2 & \text{measurement,}\\ 2\epsilon_{\max}C^2 & \text{detection,}\end{cases}$$
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Effects of Calibration Error

- Let $v(f)$ denote the raw detector output, and $R(f)$ denote the response function used to convert $v(f)$ to the observed gravitational wave signal: $h(f) = R(f)v(f)$.

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- Assume the measured response function $R(f)$ has errors that cause errors in the inferred gravitational waveform:

$$h(f) = R(f)v(f) = [R_e(f) + \delta R(f)]v(f) = h_e(f) + \delta h_R(f).$$

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- Evaluate the signal-to-noise ratio for an observed signal h with a filter based on the model waveform $h_m = h_e + \delta h_m$. Keep terms through quadratic order in δR and δh_m:

$$\rho_m = \frac{\langle h|h_m\rangle}{\langle h_m|h_m\rangle^{1/2}} = \rho - \frac{1}{2\rho}\langle(\delta h_m - \delta h_R)_\perp|(\delta h_m - \delta h_R)_\perp\rangle,$$

where $(\delta h_m - \delta h_R)_\perp = \delta h_m - \delta h_R - h_e\langle h_e|(\delta h_m - \delta h_R)\rangle/\rho^2$.

Effects of Calibration Error II

- Determine the maximum effect of response function error, δh_R, and modeling error, δh_m, on the signal-to-noise ratio ρ:

$$\begin{aligned} \rho_m &= \rho - \langle (\delta h_m - \delta h_R)_\perp | (\delta h_m - \delta h_R)_\perp \rangle / 2\rho, \\ &\geq \rho - \left[\langle \delta h_m | \delta h_m \rangle^{1/2} + \langle \delta h_R | \delta h_R \rangle^{1/2} \right]^2 / 2\rho. \end{aligned}$$

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- Define η, the ratio of model waveform error to response function error: $\langle\delta h_m|\delta h_m\rangle = \eta^2\langle\delta h_R|\delta h_R\rangle$. Re-express ρ_m as,

$$\rho_m \geq \rho - (1+\eta)^2\langle\delta h_R|\delta h_R\rangle/2\rho.$$

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- Waveform model errors less than maximum fraction of the calibration error, $\eta \leq \eta_{\max}$, are swamped by calibration error.
- Natural choices for $\eta_{\max}$ are $\eta_{\max} = 1$, or $\eta_{\max} = \sqrt{2} - 1 \approx 0.4$.

Summary of Model Waveform Accuracy Standards

- The basic model waveform accuracy standards are:

$$\frac{\langle\delta h|\delta h\rangle}{\rho^2} = \overline{\delta\chi_m}^2 + \overline{\delta\Phi_m}^2 < \begin{cases} 1/\rho^2 & \text{measurement,} \\ 2\epsilon_{\text{max}} & \text{detection.} \end{cases}$$

- Simpler conditions that guarantee the basic standards are:

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- The basic waveform accuracy standards need not be enforced when they are stricter than the response-function error condition:

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- To use these standards, we must determine the ranges for the quantities ρ, $\epsilon_{\max}$, C, $\eta_{\max}$, $\overline{\delta\chi_R}$, and $\overline{\delta\Phi_R}$ appropriate for LIGO.

Measurement Standards for LIGO

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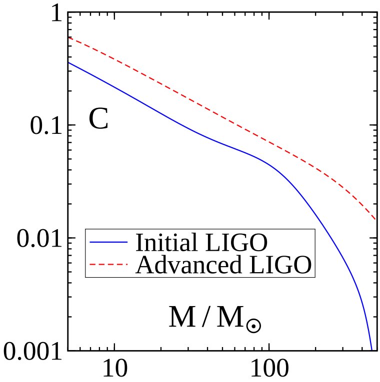
- The signal-to-noise quantity $C^2 = \rho^2 \min S_n/2||h_e||^2 \leq 1$ has been evaluated for equal-mass non-spinning black hole binaries using LIGO noise curves.



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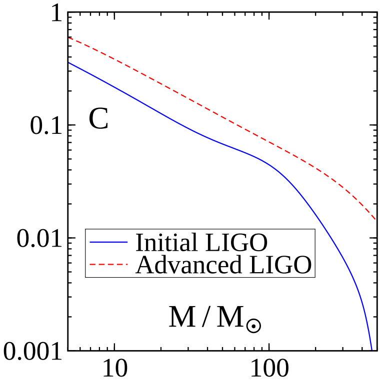

- The accuracy requirement for BBH waveforms for Advanced LIGO measurements is therefore:

$$\frac{||\delta h_m(t)||}{||h_e(t)||} < \frac{0.014}{80} \approx 0.0002 \lesssim \frac{C}{\rho}.$$

Detection Standards for LIGO

- Accuracy requirement for detection depends on the parameter ϵ_{max}, the maximum allowed mismatch between an exact waveform and its model counterpart.
- The maximum mismatch is chosen to assure searches miss only a small fraction of real signals. The common choice $\epsilon_{\text{max}} = 0.035$ limits the loss rate to about 10%.

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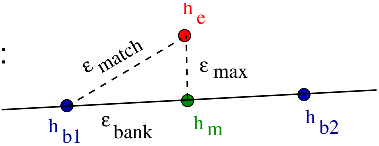

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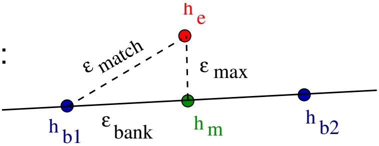

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- For Initial LIGO, template banks are constructed with $\epsilon_{\mathrm{bank}} = 0.03$, so $\epsilon_{\mathrm{max}} = 0.035 - 0.03 = 0.005$ is the appropriate choice.
- Accuracy requirement for BBH waveforms for detection in LIGO:

$$\frac{||\delta h_m(t)||}{||h_e(t)||} < \sqrt{2\epsilon_{\mathrm{max}}}C = \sqrt{2 \times 0.005} \times 0.014 \approx 0.0014.$$

Calibration Error Effects in LIGO

- Model waveform errors are insignificant when they are smaller than some fraction ($\eta_{\rm max} \lesssim 0.4$) of the response function error,
$$\overline{\delta\chi_m}^2 + \overline{\delta\Phi_m}^2 \leq \eta_{\rm max}^2 \left[\overline{\delta\chi_R}^2 + \overline{\delta\Phi_R}^2\right].$$

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- For the Initial LIGO S4 data, the calibration errors are:

$$0.03 \le \sqrt{\delta\chi_R^2 + \delta\Phi_R^2} \le 0.09 \qquad \text{L1}$$
$$0.06 \le \sqrt{\delta\chi_R^2 + \delta\Phi_R^2} \le 0.12 \qquad \text{H1}$$

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- A sufficient condition for model waveform error to be insignificant compared to calibration error is therefore:

$$\begin{aligned} \frac{||\delta h_m(t)||}{||h_e(t)||} &\le \eta_{\rm max} C \sqrt{\min|\delta\chi_R|^2 + \min|\delta\Phi_R|^2} \\ &= 0.4 \times 0.014 \times 0.03 \approx 0.0002. \end{aligned}$$

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- Model waveform errors are insignificant when they are smaller than some fraction ($\eta_{\max} \lesssim 0.4$) of the response function error,

$$\overline{\delta\chi_m}^2 + \overline{\delta\Phi_m}^2 \leq \eta_{\max}^2 \left[\overline{\delta\chi_R}^2 + \overline{\delta\Phi_R}^2\right].$$

- For the Initial LIGO S4 data, the calibration errors are:

$$\begin{aligned} 0.03 &\leq \sqrt{\delta\chi_R^2 + \delta\Phi_R^2} \leq 0.09 && \mathrm{L1} \\ 0.06 &\leq \sqrt{\delta\chi_R^2 + \delta\Phi_R^2} \leq 0.12 && \mathrm{H1} \end{aligned}$$

- A sufficient condition for model waveform error to be insignificant compared to calibration error is therefore:

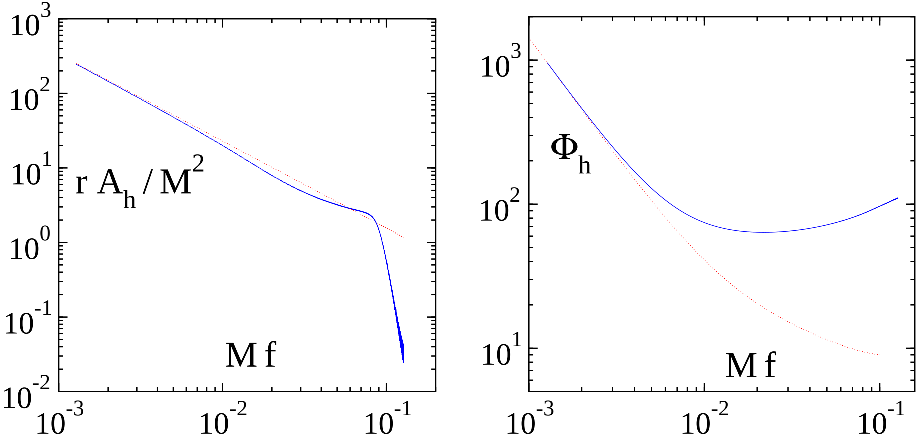
$$\begin{aligned} \frac{\|\delta h_m(t)\|}{\|h_e(t)\|} &\leq \eta_{\max} C \sqrt{\min|\delta\chi_R|^2 + \min|\delta\Phi_R|^2} \\ &= 0.4 \times 0.014 \times 0.03 \approx 0.0002. \end{aligned}$$

- The ideal-detector measurement standard requires waveform errors smaller than calibration errors for $\rho \gtrsim 80$, so calibration errors prevent optimal accuracy measurements for these sources.

The End

Extra Slides for Discussion

Frequency Domain BBH Waveforms (Equal Mass Non-Spinning)



FFT of BBH waveform from Scheel, et al. (2008).

- Summary

Waveform Error Diagnostic	Measurement Requirement	Detection Requirement
$\overline{\delta\Phi}$	$1/\sqrt{2}\,\rho$	$\sqrt{\epsilon_{\rm max}}$
$\max\|\delta\Phi\|$	$1/\sqrt{2}\,\rho$	$\sqrt{\epsilon_{\rm max}}$
$\|\|\delta h(t)\|\|/\|\|h_e(t)\|\|$	C/ρ	$\sqrt{2\epsilon_{\rm max}}\,C$

- LIGO

Waveform Error Diagnostic	Measurement Requirement	Detection Requirement
$\overline{\delta\Phi}$	0.009	0.07
$\max\|\delta\Phi\|$	0.009	0.07
$\|\|\delta h(t)\|\|/\|\|h_e(t)\|\|$	0.0002	0.001

- LISA

Waveform Error Diagnostic	Measurement Requirement	Detection Requirement
$\overline{\delta\Phi}$	$4 \cdot 10^{-5}$	0.07
$\max\|\delta\Phi\|$	$4 \cdot 10^{-5}$	0.07
$\|\|\delta h(t)\|\|/\|\|h_e(t)\|\|$	$3 \cdot 10^{-7}$	0.0005