Use and Abuse of the Model Waveform Accuracy Standards

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- Accuracy standards should be imposed on model waveform and detector calibration accuracies:
  - to prevent a significant rate of missed detections,
  - to prevent accuracy losses in measurements,
  - to avoid unnecessary costs of achieving excess accuracy.
- This talk will describe possible abuses of the standards, and ways to avoid them.

Waveform and Calibration Accuracy Standards:

- Combined waveform and calibration accuracy standards:

$$\sqrt{\langle \delta h_m | \delta h_m \rangle} + \sqrt{\langle \delta h_R | \delta h_R \rangle} < \begin{cases} 1 & \text{measurement,} \\ \rho\sqrt{2\epsilon_{\max}} & \text{detection,} \end{cases}$$

  - $\delta h_m = h_m - h_e$ Model waveform error.
  - δh_R Errors from calibration inaccuracies.

  - Standards are written in terms of the noise-weighted inner product:

$$\langle h_e | h_m \rangle = 2\int_0^\infty \frac{h_e^*(f) h_m(f) + h_e(f) h_m^*(f)}{S_n(f)} df,$$

    where $S_n(f)$ is the power spectral density of the detector noise.

  - The maximum allowed errors are determined by ρ, the signal to noise ratio, and $\epsilon_{\max}$ which determines the missed detection loss rate (typically set to $\epsilon_{\max} = 0.005$).

More Intuitive Waveform Accuracy Standards

- Waveform accuracy standards can be re-written as:

$$\frac{\sqrt{\langle \delta h_m | \delta h_m \rangle}}{\rho} = \sqrt{\overline{\delta\chi_m}^2 + \overline{\delta\Phi_m}^2} < \begin{cases} 1/(2\rho_{\max}) & \text{measurement,} \\ \sqrt{2\epsilon_{\max}} & \text{detection.} \end{cases}$$

  - Amplitude $\delta\chi_m$ and phase $\delta\Phi_m$ errors are defined as $\delta h_m = h_e e^{\delta\chi_m + i\delta\Phi_m} - h_e \approx h_e(\delta\chi_m + i\delta\Phi_m)$.
  - Signal-weighted average errors are defined as

$$\overline{\delta\chi_m}^2 = \int_0^\infty \delta\chi_m^2 \frac{4|h_e|^2}{\rho^2 S_n} df, \quad \text{and} \quad \overline{\delta\Phi_m}^2 = \int_0^\infty \delta\Phi_m^2 \frac{4|h_e|^2}{\rho^2 S_n} df.$$

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- How do you relate $\delta\chi_m(f)$ and $\delta\Phi_m(f)$ to the time-domain waveform errors that arise in waveform modeling?
- How do you estimate these errors reliably?

Maximum Error Fallacy

- Some NR groups have estimated the maximum time-domain waveform errors $\max|\delta\chi_t|$ and $\max|\delta\Phi_t|$, and compared them with the standards for $|\overline{\delta\chi_m}|$ and $|\overline{\delta\Phi_m}|$.
- Is this good enough?

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- Consider a model waveform: $h_m(t)$ with errors of the form:

$$h_m(t) = A_e(t)\Big[1 + {}_{\max}|\delta\chi_t|\, g_\chi(t)\Big]\cos\Big[\Phi_e(t) + {}_{\max}|\delta\Phi_t|\, g_\Phi(t)\Big],$$

with $g_\chi = g_\Phi = \cos[\lambda\Phi_e(t)]$.

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- Compute ratio of frequency- to time-domain error measures,

$$R = \sqrt{\frac{\overline{\delta\chi_m}^2 + \overline{\delta\Phi_m}^2}{\max(|\delta\chi_t|^2 + |\delta\Phi_t|^2)}}$$

using the PN+Caltech/Cornell waveform for A_e and Φ_e.

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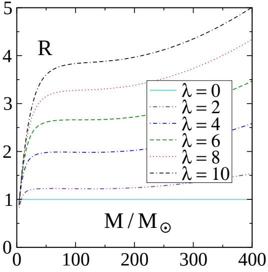

- Bad News! Limiting $\max|\delta\chi_t|$ and $\max|\delta\Phi_t|$ is not sufficient.

Error Envelope Fallacy

- Additional knowledge of the full waveform errors, $\max|\delta\chi_t|\, g_\chi(t)$ and $\max|\delta\Phi_t|\, g_\Phi(t)$, is needed. Unfortunately the exact time dependencies, $g_\chi(t)$ and $g_\Phi(t)$, will never be known.
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- Is a partial knowledge of $g_\chi(t)$ and $g_\Phi(t)$ sufficient?
- Probably the most we will ever know will be local-in-time error envelope-functions $G_\chi(t)$ and $G_\Phi(t)$, that satisfy

$$|g_\chi(t)| \leq G_\chi(t) \leq 1, \quad \text{and} \quad |g_\Phi(t)| \leq G_\Phi(t) \leq 1.$$

- Do time-domain bounds imply frequency-domain bounds, i.e., does $|g(t)| \leq G(t)$ imply $|g(f)| \leq G(f)$?

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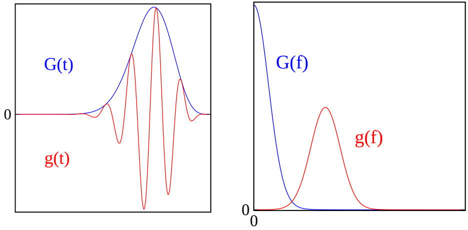
- Do time-domain bounds imply frequency-domain bounds, i.e., does $|g(t)| \leq G(t)$ imply $|g(f)| \leq G(f)$?
- No!
- It is not possible to verify the accuracy of a waveform using a time-domain error-envelope function.



Time Domain Accuracy Standards

- An alternate form of the accuracy standards can be written in terms of the time domain L^2 norm $||\delta h_m(t)||^2 = \int_{-\infty}^{\infty} |\delta h_m|^2 dt$.
- This alternate standard has the form:

$$\frac{||\delta h(f)||}{||h_m(f)||} = \frac{||\delta h(t)||}{||h_m(t)||} < \frac{C}{2\rho},$$

where C, is a scale invariant ratio of two signal-to-noise measures

$$C^2 = \frac{\rho^2}{2||h_m(f)||^2/{}_{\min}S_n(f)} \leq 1.$$

- The error envelope functions, ${}_{\max}|\delta\chi_t|\, G_\chi(t)$ and ${}_{\max}|\delta\Phi_t|\, G_\Phi(t)$, provide strict upper limits for these error measures.

Summary and Questions

- Combined accuracy standards now exist for waveform accuracy and calibration. The model waveform standards can be written as:

$$\sqrt{\overline{\delta\chi_m}^2 + \overline{\delta\Phi_m}^2} < \begin{cases} 1/(2\rho_{\max}) & \text{measurement,} \\ \sqrt{2\epsilon_{\max}} & \text{detection.} \end{cases}$$

- The basic standards are difficult (impossible?) to enforce directly, so easier to enforce time-domain conditions have been derived:

$$\frac{||\delta h_m(t)||}{||h_m(t)||} \leq \sqrt{\frac{\int_{-\infty}^{\infty} A_m^2 \left(\max|\delta\chi_t|^2 G_\chi^2 + \max|\delta\Phi_t|^2 G_\Phi^2\right) dt}{\int_{-\infty}^{\infty} A_m^2 dt}} \lesssim \begin{cases} C/(2\rho_{\max}) \\ C\sqrt{2\epsilon_{\max}} \end{cases}$$

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- How well do the calibration and search template accuracies currently being used by LIGO satisfy these requirements?
- How well do the waveforms produced by various NR groups satisfy these requirements?