Gravitational Radiation Instabilities in Rotating Neutron Stars*

Lee Lindblom

Theoretical Astrophysics 130-33,
California Institute of Technology,
Pasadena, CA 91125 (USA)

Abstract

Gravitational radiation (GR) drives an instability in certain modes of rotating stars. This instability is strong enough in the case of the r-modes to cause their amplitudes to grow (in the absence of other dissipation) on a timescale of tens of seconds in rapidly rotating neutron stars. GR emitted by these modes removes angular momentum from the star at a rate which would spin it down to a relatively small angular velocity within about one year, if the dimensionless amplitude of the mode grows to order unity. A pedagogical discussion is given here of the mechanism of GR instability in rotating stars, on the relevant properties of the r-modes, and on our present understanding of the dissipation mechanisms (including interactions with the crust and hyperon bulk viscosity) that tend to suppress this instability in neutron stars. The astrophysical implications of this GR driven instability are discussed for young neutron stars, and for older systems such as low mass x-ray binaries. Recent work on the evolution and saturation of the r-modes by non-linear hydrodynamic effects is also described.

1 Introduction

The non-radial pulsations of stars couple to gravitational radiation (GR) in general relativity theory [2, 3], and the GR produced by these oscillations carries away energy and angular momentum from the star. In non-rotating stars the effect of these GR losses is dissipative, and the pulsations of the star are damped. Chandrasekhar first noted [4, 5] that in rotating stars the situation can be quite different: the emission of GR causes the amplitudes of certain modes to grow. The mechanism that drives this GR instability is fairly easy to understand: Modes that propagate in the direction opposite the star's rotation (as seen in the co-rotating frame of the fluid) have *negative* angular momentum, because these modes lower the total angular momentum of the star. In a rotating star some of these counter-rotating modes are dragged forward by the rotation of the star and appear to an inertial observer to propagate in the same direction as the star's rotation. Such modes, as illustrated in Fig. 1, emit *positive* angular momentum GR since the density and momentum perturbations appear to an observer at infinity to be rotating in the same direction as the star. The angular momentum removed by GR lowers the (already negative) angular momentum of such a mode, and therefore the amplitude of the mode grows.

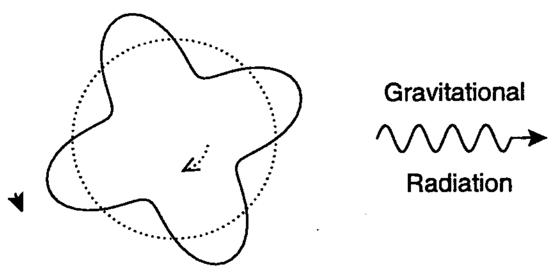

Figure 1: A counter-rotating mode (solid curve) that is dragged forward by the rotation of the background star (dashed curve) is driven unstable by the emission of gravitational radiation.

*The introductory portions of this review are based very closely on Ref. [1].

This GR driven instability was first studied extensively by Friedman and Schutz [6, 7] for the fundamental (f-) modes of rotating stars. They demonstrated that the GR instability has the remarkable property that it makes *every* rotating perfect fluid star unstable in general relativity theory. This discovery sparked an interest in the possibility that GR might play a significant role in the evolution of real neutron stars. Does the GR instability determine the maximum spin rate of pulsars? Is the GR emitted by an unstable rapidly rotating neutron star detectable? Unfortunately the generic nature of this destabilizing process does not guarantee that it plays any role at all in real neutron stars. Internal dissipation (*e.g.,* viscosity) within a star tends to damp the pulsations that are driven unstable by GR. If the internal dissipation is sufficiently strong, then the GR instability can even be suppressed completely [8, 9]. Detailed calculations of the effects of GR and internal dissipation on the f-modes of rotating stars revealed that the GR instability is effective only in very rapidly rotating stars [10, 11, 12, 13]. Stars with angular velocities smaller than some critical value, $\Omega < \Omega_c$ are stable, while those rotating more rapidly, $\Omega > \Omega_c$, are subject to the GR instability. This critical angular velocity, Ω_c, for the stability of the f-modes is depicted in Fig. 2 for realistic neutron-star models. The strength of the internal dissipation processes in neutron stars is temperature dependent, and consequently the critical angular velocity Ω_c is temperature dependent as well. Figure 2 illustrates that the GR instability is completely suppressed in the f-modes except when the temperature of the neutron star lies in the range, $10^7 < T < 10^{10}$K. Further, the internal dissipation is so strong that the f-modes are never unstable unless the angular velocity of the star exceeds $0.91\Omega_{\rm max}$. Thus the GR instability in the f-modes can not significantly reduce the spin of a neutron star below the maximum, and substantial amounts of GR can not be emitted by this process.

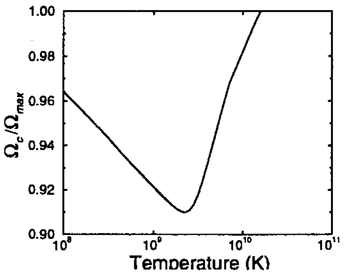

Figure 2: Temperature dependence of the critical angular velocity Ω_c in rotating neutron stars: an f-mode is driven unstable by gravitational radiation when the star's angular velocity exceeds Ω_c.

This pessimistic view of the GR instability began to change when Andersson [14] and Friedman and Morsink [15] showed that the r-modes were also subject to the GR instability. Indeed they showed that *all* the r-modes are driven unstable by GR in *all* rotating perfect fluid stars. Subsequent calculations by Lindblom, Owen and Morsink [16] showed that the GR instability in the r-modes was also strong enough to overcome the simplest internal dissipation processes in neutron-star matter, even in relatively slowly rotating stars. Thus the GR instability in the r-modes is strong enough that it might be capable of significantly reducing the angular momenta of rotating neutron stars, and the GR emitted during such spin-down events might perhaps be detectable by LIGO [17]. The remainder of this paper discusses recent developments related to the GR instability of the r-modes. Section 2 discusses the basic properties of the r-modes and their GR instability. Section 3 reviews the astrophysical scenarios in which the r-mode GR instability might play an important role. This discussion focuses on recent work that evaluates the effects of the neutron-star crust, hyperon bulk viscosity, and non-linear hydrodynamics on the r-mode instability. Taken together these various effects now make it seem rather unlikely that the r-mode instability plays an important role in any of the proposed astrophysical scenarios. Section 4 summarizes some of the open questions that prevent us at this time from knowing for certain whether the r-mode instability plays an important role in real astrophysical systems.

2 Gravitational Radiation Instability in the r-Modes

The r-modes (also called rotation dominated modes, inertial modes, or Rossby waves) are oscillations of rotating stars whose restoring force is the Coriolis force [18]. These modes are primarily velocity perturbations, which for slowly rotating barotropic stars have the simple analytical form

$$\delta\vec{v} = \alpha R\Omega \left(\frac{r}{R}\right)^m \vec{Y}^B_{mm} e^{i\omega t} + \mathcal{O}(\Omega^3), \tag{1}$$

where α is the dimensionless amplitude of the mode; R and Ω are the radius and angular velocity of the equilibrium star; $\vec{Y}^B_{lm} = \hat{r} \times r\vec{\nabla}Y_{lm}/\sqrt{l(l+1)}$ is the magnetic-type vector spherical harmonic; and ω is the frequency of the mode. The associated density perturbation, $\delta\rho = \mathcal{O}(\Omega^2)$, vanishes at lowest order. Because the Coriolis force dominates, the frequencies of the r-modes are independent of the equation of state and are proportional to the angular velocity of the star (at lowest order),

$$\omega = -\frac{(m-1)(m+2)}{m+1}\Omega + \mathcal{O}(\Omega^3). \tag{2}$$

The velocity field of the r-mode, Eq. (1), is everywhere orthogonal to the radial direction $\hat{r}$, and has an angular structure determined by Y_{mm}. Figure 3 gives equatorial and polar views of this velocity field for the $m = 2$ r-mode, which plays the dominant role in the GR instability. Figure 4 shows another view of the same field in standard polar coordinates (θ, φ). The four circulation zones propagate through the fluid with angular velocity $-\frac{1}{3}\Omega$, toward the left in Fig. 4. The fluid elements respond by moving on paths described by the Lagrangian displacement, $\vec{\xi} = -i\delta\vec{v}/(\omega + m\Omega)$. To first order these are ellipses, with θ-dependent eccentricities, as illustrated on the left side of Fig. 4.

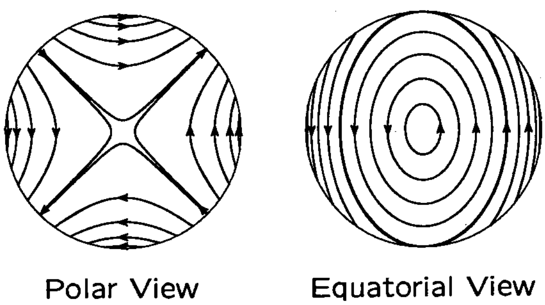

Figure 3: Polar and equatorial views of the flow pattern of the $m = 2$ r-mode. This velocity field propagates through the fluid with angular velocity $\frac{2}{3}\Omega$ relative to the inertial frame, and $-\frac{1}{3}\Omega$ relative to the fluid.

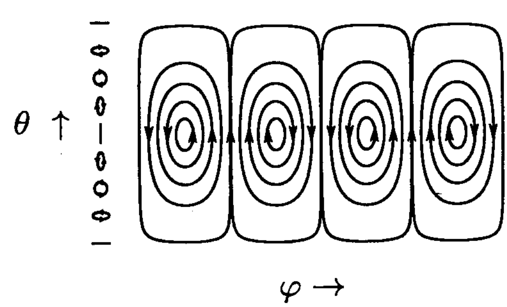

Figure 4: Polar coordinate (θ, φ) representation of the flow pattern of the $m = 2$ r-mode. The pattern moves past the individual fluid elements which respond by moving on small elliptical paths as illustrated on the left.

The effects of dissipation (*i.e.*, viscosity and GR) on the evolution of the r-mode are most easily studied by considering $\tilde{E}$, the energy of the perturbation (as measured in the co-rotating frame of the fluid). To lowest order in Ω, $\tilde{E}$ is given by

$$\tilde{E} = \frac{1}{2}\int \rho\,\delta\vec{v}^* \cdot \delta\vec{v}\, d^3x + \mathcal{O}(\Omega^4). \tag{3}$$

This energy is conserved in the absence of dissipation, and more generally satisfies [16]

$$\frac{d\tilde{E}}{dt} = -\omega(\omega + m\Omega)\sum_{l\geq m} N_l\,\omega^{2l}\left[|\delta D_{lm}|^2 + \frac{4l|\delta J_{lm}|^2}{c^2(l+1)}\right] - \int \left(2\eta\,\delta\sigma^*_{ab}\delta\sigma^{ab} + \zeta\,\delta\sigma^*\delta\sigma\right) d^3x, \tag{4}$$

where $N_l = 4\pi G(l+1)(l+2)\{c^{2l+1}l(l-1)[(2l+1)!!]^2\}^{-1}$ are positive constants; and δD_{lm} and δJ_{lm} are the mass and current multipole moments of the perturbation,

$$\delta D_{lm} = \int \delta\rho\, r^l Y^*_{lm} d^3x, \tag{5}$$

$$\delta J_{lm} = \int r^l\,(\rho\,\delta\vec{v} + \delta\rho\,\vec{v})\cdot \vec{Y}^{B*}_{lm}\, d^3x. \tag{6}$$

The second term on the right side of Eq. (4) represents the dissipation due to the shear and bulk viscosity of the fluid: η and ζ are the viscosity coefficients, and $\delta\sigma^{ab}$ and $\delta\sigma$ are the shear and expansion of the perturbed fluid respectively. These viscosity terms in Eq. (4) always decrease the energy $\tilde{E}$ and so tend to damp the r-modes. The first term on the right side of Eq. (4) represents the effect of GR on the perturbation. The sign of this term is determined by the sign of $\omega(\omega + m\Omega)$, the product of the frequencies in the inertial and rotating frame. This product,

$$\omega(\omega + m\Omega) = -\frac{2(m-1)(m+2)}{(m+1)^2}\Omega^2 < 0, \tag{7}$$

is negative for the r-modes, thus GR tends to drive the r-modes toward instability. Further this destabilizing force is *generic* [14, 15]: GR drives all the r-modes in all rotating stars (*i.e.*, for all values of m and Ω) toward instability.

To evaluate the relative strengths of the destabilizing GR force and the dissipative viscous forces, it is convenient to define the combined dissipative timescale $1/\tau$,

$$\frac{1}{\tau} = -\frac{1}{2\tilde{E}}\frac{d\tilde{E}}{dt} = -\frac{1}{\tau_{GR}} + \frac{1}{\tau_V}, \tag{8}$$

which is just the imaginary part of the frequency of the mode. The integrals on the right sides of Eqs. (3)–(6) are easily performed to determine the GR and the viscous contributions to $1/\tau$ respectively. Using Newtonian stellar models based on fairly realistic neutron-star matter these timescales are [16, 19]:

$$\frac{1}{\tau_{GR}} = \frac{1}{38\text{s}}\left(\frac{\Omega}{\Omega_{\max}}\right)^6, \tag{9}$$

$$\frac{1}{\tau_V} = \frac{1}{3\times 10^8\text{s}}\left(\frac{10^9\text{K}}{T}\right)^2 + \frac{1}{5\times 10^{11}\text{s}}\left(\frac{T}{10^9\text{K}}\right)^6\left(\frac{\Omega}{\Omega_{\max}}\right)^2. \tag{10}$$

For small angular velocities, $\Omega \ll \Omega_{\max}$, the GR timescale is very large so viscous dissipation always dominates, $1/\tau_{GR} \ll 1/\tau_V$. Thus neutron stars are always stable in this limit. Conversely, when Ω is sufficiently large the GR timescale is shorter than the viscous timescale and the neutron star is unstable. The critical angular velocity Ω_c,

$$\frac{1}{\tau(\Omega_c)} = 0, \tag{11}$$

marks the boundary between stability and instability. Since the viscosities are temperature dependent in neutron-star matter, so too is Ω_c. The solid curve in Fig. 5 illustrates the temperature dependence of Ω_c for the r-modes including the effects of standard microscopic shear and bulk viscosity. The minimum of this curve occurs at $\min \Omega_c = 0.045\Omega_{\max}$. For comparison Fig. 5 also illustrates Ω_c for the GR instability in the f-modes. It is obvious that GR is capable of driving the r-modes unstable over a far wider range of angular velocities than the f-modes. Thus the GR instability in the r-modes might play an interesting role in limiting the angular velocities of neutron stars, and the GR emitted during a spin-down event might be detectable. We will return to a more in depth discussion of some more realistic dissipation mechanism which may effect the r-mode instability after introducing the principal scenarios where the instability may play an interesting role in real astrophysical systems.

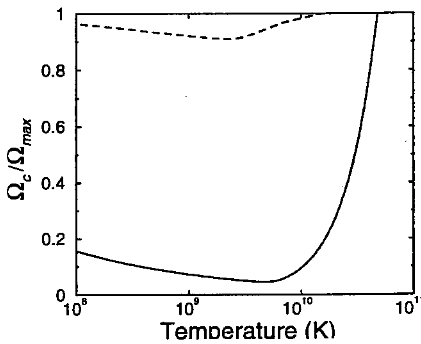

Figure 5: Temperature dependence of the critical angular velocity Ω_c for rotating neutron stars. Solid curve gives Ω_c for the instability in the $m = 2$ r-mode, while for comparison the dashed curve gives Ω_c for the f-modes.

3 Astrophysical Implications

Two astrophysical scenarios have been proposed in which the GR instability of the r-modes might play an interesting role in the evolution of real neutron stars. These are illustrated by the two evolution curves, A and B, in Fig. 6. In scenario A a rapidly rotating neutron star is formed with a very high temperature ($T \geq 10^{11}$K) as the result of the gravitational collapse of the neutron-star progenitor [16]. In this scenario the star cools within a few seconds to a point that lies above the r-mode instability curve (the dashed curve in Fig. 6). The amplitude of the r-mode then grows exponentially (with a timescale of about 40 s for a very rapidly rotating star), and becomes large within a few minutes. If the dimensionless r-mode amplitude α saturates (by some yet to be understood process) with a value of order unity, it would take about 1 y for the star to spin down to a point where stability is re-gained [17]. In this scenario a star could lose up to 95% of its angular momentum, and up to about 99% of its rotational kinetic energy by emitting GR. This scenario might provide a natural explanation for the lack of rapidly rotating neutron stars in young supernova remnants. The GR emitted in this scenario might be detectable for neutron stars as far away as the Virgo cluster [17, 20].

In scenario B an old, cold slowly rotating neutron star is spun up by accreting high angular momentum material from a companion star [21, 22]. Once the neutron star's angular velocity reaches the critical value Ω_c, the amplitude of the unstable r-mode grows exponentially. It was once thought that in this situation the amplitude of the unstable mode would grow until the rate of angular momentum lost to GR just balances the amount gained from accretion [23]. However Levin [24] has shown that viscous dissipation in the growing r-mode rapidly increases the temperature of the low specific-heat neutron-star matter. This moves the star along the horizontal section of the evolution curve B in Fig. 6. At some point the r-mode amplitude saturates (by some yet to be understood mechanism) and thermal equilibrium is established between viscous heating and neutrino cooling. The star then spins down by emitting GR until stability is regained. It has been suggested that this scenario provides an explanation for the range of rotation periods observed for the neutron stars in low mass x-ray binaries (LMXBs) [25].

These scenarios are just rough sketches and considerable work has been (and continues to be) done to fill in the details and see whether they represent realistic astrophysical possibilities. In the case of scenario B for example, it is clear that the sketch given above is too simple. The core temperatures of neutron stars in accreting systems like the LMXBs are expected to be in the range $10^8 - 10^9$K [26, 27].

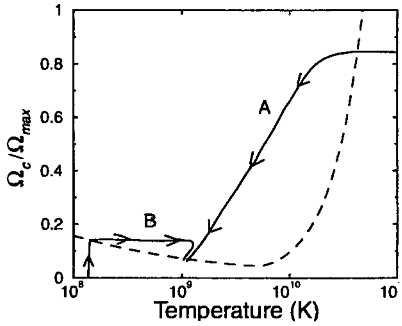

Figure 6: Rotating neutron stars may become unstable to the r-mode instability in two ways: *a)* hot young rapidly rotating stars may cool along path A, become unstable, and finally spin down to a small angular velocity; or *b)* old cold slowly rotating stars may be spun up by accretion along path B, becoming unstable, then heated by the growing r-mode, and finally spun down to a smaller angular velocity.

Simple shear viscosity gives rise to $\Omega_c \leq 0.16\Omega_{\max}$ in this temperature range, as seen in Fig. 5. This upper limit (of about 160 Hz) on the angular velocities of accreting systems is in conflict with the observed 300 Hz spin frequencies of the neutron stars in LMXBs, and the 600 Hz frequencies of pulsars that are believed to have been spun up in LMXB-like systems. Thus some additional dissipation mechanism must act to suppress the r-mode instability in these accreting systems. It was suggested [16] that additional dissipative effects associated with the superfluid transition in the neutron-star matter at about 10^9K might effectively suppress the r-mode instability. However, the calculations done to date indicate that the dominant superfluid dissipation mechanism (mutual friction) is generally not effective in suppressing the r-mode instability [28]. Bildsten and Ushomirsky [29] have suggested that viscous dissipation in the boundary layer between the liquid core and the solid crust of a neutron star might provide the needed stability. And Mendell [30] has shown that dissipation associated with Alfven waves (which are excited by the r-modes and then travel along magnetic field lines that are pinned to the solid crust) could in some circumstances be even stronger than the viscous boundary layer dissipation. At present it appears that some combination of these crust, magnetic field, and superfluid effects are the most likely candidates for stabilizing the r-modes in this temperature range.

At the interface between a viscous fluid and a solid (*e.g.*, the crust of a neutron star) the fluid velocity must match the velocity of the solid. Therefore viscosity significantly modifies the velocity field of an r-mode, at least in the neighborhood of the crust-core boundary. The solution of the viscous fluid equations in this boundary region [29, 33, 34] shows that the r-mode velocity field is modified significantly only in a thin layer with scale-height d,

$$d = \sqrt{\frac{\eta}{2\rho\Omega}} \approx 0.6\text{cm}\left(\frac{10^9\text{K}}{T}\right)\left(\frac{\Omega_{\max}}{\Omega}\right)^{1/2}. \tag{12}$$

The magnitude of the shear of the fluid in this boundary layer is approximately $|\delta\sigma^{ab}| \approx |\vec{\nabla}\delta\vec{v}| \approx |\delta\vec{v}|/d$, which is larger by the factor $R/d \approx 10^6$ than the shear of the inviscid r-mode velocity field. The formation of a rigid crust therefore increases the total dissipation due to shear viscosity by approximately the factor R/d. The viscous timescale for the $m = 2$ r-mode (using a typical neutron-star model) then becomes [34],

$$\tau_V = \begin{pmatrix} 280\,\text{s}, & T < 10^9\text{K} \\ 650\,\text{s}, & T > 10^9\text{K} \end{pmatrix}\left(\frac{T}{10^9\text{K}}\right)\left(\frac{\Omega_{\max}}{\Omega}\right)^{1/2}. \tag{13}$$

Figure 7 illustrates the critical angular velocity Ω_c for the r-mode GR instability including the effects of this boundary-layer dissipation. The solid curves are based on neutron-star models from a number of realistic equations of state. Figure 7 illustrates that dissipation in the boundary layer significantly increases the stability of the r-modes. This suggests that rapidly rotating neutron stars, such as the 1.6 ms pulsars, are consistent with a spin-up process that operates in the $10^8 - 10^9$ K temperature range. And this suggests that the apparent clustering of spin frequencies in the LMXBs is probably not due to the GR instability in the r-modes. However, additional work is needed to understand fully whether scenario B ever operates in real neutron stars or not. In particular the effects of a semi-rigid crust (which tend to reduce the boundary layer dissipation) [35] have not been included in Fig. 7, nor have the effects of the neutron star's magnetic field [36, 37, 30], nor have other possible effects of the superfluid core (*e.g.*, the possibility that the core vortices are pinned to the crust, or a possible dissipative interaction between neutron vortices and magnetic flux tubes).

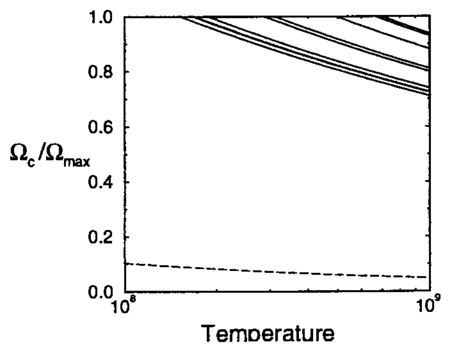

Figure 7: Solid curves represent Ω_c for neutron star models (from a variety of realistic equations of state) with rigid crust. Dashed curve represents the stability curve for a neutron star without crust.

Let us turn our attention now to astrophysical scenario A. At present it also seems unlikely that this scenario will play an interesting role in the astrophysics of real neutron stars. This pessimistic view is based on our current understanding of the effects of several complicated physical mechanisms on the r-modes. In particular the formation of a solid crust, exotic forms of microscopic dissipation in the fluid, and non-linear hydrodynamic effects are all now expected to reduce in a significant way the importance of the r-mode instability in scenario A. Here I review briefly what is known at present about how each of these processes effects the r-modes. Let us consider first the role of a solid crust. When a neutron star cools to about 10^{10}K (within about 30 s after its birth according to the standard modified URCA cooling model, or within as little as a fraction of a second according to cooling calculations that include the direct URCA process [31, 32]), a solid crust begins to form initially at densities of about $\rho_c \approx 1.5 \times 10^{14}\text{gm/cm}^3$ [34]. Figure 8 shows typical values for the critical angular velocity Ω_c (the dashed curve) based on the boundary-layer dissipation from a rigid crust in the temperature range that is relevant for scenario A. If cooling proceeds by some rapid mechanism, such as direct URCA (as suggested by recent observations [38]), stars rotating slower than $\sim 0.8\Omega_{max}$ could never develop significant r-mode amplitudes: the star would cool into the stable range before the instability had time to grow. And even if the slower modified URCA cooling dominates in the portions of the star where the crust will form, only stars rotating faster than $\sim 0.5\Omega_{max}$ could develop a significant r-mode instability.

If a neutron star is born with a large angular velocity, then it may still be subject to the r-mode instability even in the presence of a rigid crust. In this case fluid flow in the boundary layer soon becomes turbulent as the amplitude of the r-mode grows. Under these conditions turbulent viscosity significantly increases the dissipation at the crust-core boundary. Wu, Matzner, and Arras [39] have shown in this case that non-linearities in the energy dissipation rate cause the amplitude of the r-mode to saturate at the value $\alpha_{sat} \approx 0.002(\Omega/\Omega_{max})^5$. In very rapidly rotating stars, $\Omega > 0.87\Omega_{max}$, this amplitude is large enough that dissipation in the boundary layer can re-melt the crust [34]. But in more slowly rotating stars the mode saturates by this turbulent viscosity mechanism before the critical melting amplitude is reached. In these cases, $\Omega < 0.87\Omega_{max}$, it appears that the crust prevents the r-mode from growing large enough to significantly change the angular momentum of the neutron star before the star cools into a region where the r-modes are no longer unstable.

Another effect that may play an important role in the r-mode stability is a form of bulk viscosity caused by hyperons in the neutron-star core [40, 41, 42]. At densities which are found in the cores of realistic neutron-star models, Σ^- and Λ hyperons probably exist in β-equilibrium with the neutrons and protons. When a fluid perturbation (such as an r-mode) changes the density of this material, weak interactions will attempt to adjust the concentrations of the hyperons to re-establish β-equilibrium. However these

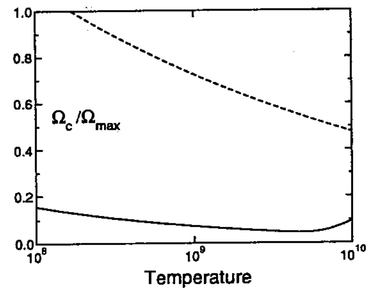

Figure 8: Curves representing the critical angular velocity Ω_c for the r-mode instability: solid curve assumes no crust has formed, dashed curve assumes standard viscous boundary layer dissipation with a rigid crust.

weak interactions are rather slow, and so equilibrium is not re-established instantaneously. There results a phase lag between the physical pressure perturbation in the material, and the appropriate equilibrium-state pressure for fluid at the density associated with the perturbation. This pressure mismatch causes dissipation, which may be characterized as a (frequency dependent) bulk viscosity. Figure 9 illustrates this bulk viscosity for a range of neutron-star matter densities, and for a range of temperatures relevant for neutron stars [42]. The effect of this type of bulk viscosity on the r-modes has been evaluated using the expressions in Eq. (4) and (8). Figure 10 illustrates the resulting neutron-star critical angular velocities for a range of neutron-star masses. We see that hyperon bulk viscosity completely suppresses the r-mode instability in 1.4 $M_\odot$ neutron stars for temperatures below a few times 10^9 K. Cooling calculations which include direct URCA reactions involving neutrons, protons and/or hyperons suggest that a neutron star will cool below this temperature within a matter of seconds: too rapidly to allow the r-mode amplitude to grow large enough to emit any significant amount of GR. Thus it appears likely that rapid cooling and the hyperon induced bulk viscosity (in addition to the dissipation from a crust discussed earlier) make it very difficult for the r-mode instability to play an interesting role in young neutron stars according to scenario A.

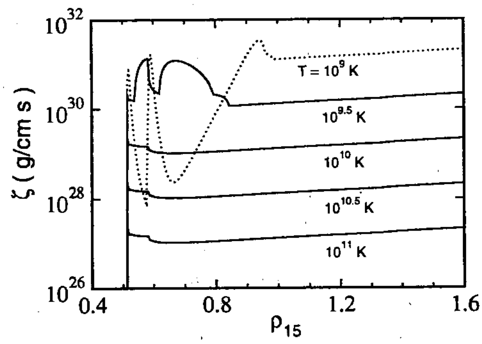

Figure 9: Density dependence (in units of 10^{15} g/cm^3) of the hyperon bulk viscosity (in units of g/cm s) for a range of temperatures.

Finally, there have been a number of recent efforts to explore the effects of non-linear hydrodynamics on the evolution of an unstable r-mode. What would happen if an r-mode in a very rapidly rotating star were somehow to escape the various dissipation mechanisms discussed above and manage (despite present expectations) to grow for several minutes so that its amplitude became large? How large could the amplitude of such an r-mode grow? What mechanism finally limits the amplitude of such a mode? There have been several large scale numerical studies, and some interesting recent analytical insights into these questions. The large-scale numerical studies consisted of putting a rapidly rotating neutron-star model

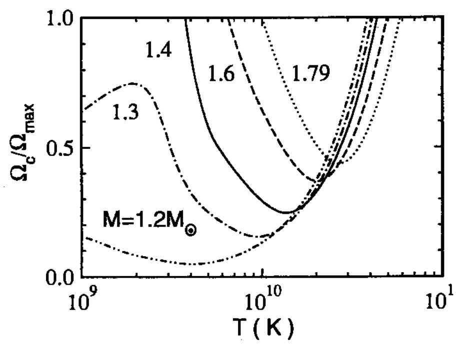

Figure 10: Critical angular velocities for neutron stars as a function of hyperon core temperature. Each curve represents a neutron star of fixed mass, ranging from $1.2M_\odot$ to the maximum mass of $1.79M_\odot$ for the equation of state used here.

with excited r-mode on a 3D numerical grid and then evolving the non-linear hydrodynamic equations to see what happens. Stergioulas and Font [43] used fully relativistic hydrodynamics in the Cowling approximation to model this system. They found that large amplitude non-linear r-modes evolve without significant dispersion in rapidly rotating fully relativistic stellar models (for tens of rotation periods). Lindblom, Tohline, and Vallisneri [44, 45] studied the growth of an unstable r-mode using Newtonian hydrodynamic equations coupled to a post-Newtonian expression for the GR reaction force. Neither study found any non-linear hydrodynamic process that prevented the amplitude of the r-mode from growing to a value of order unity. These studies however were limited by the availability of computational resources to somewhat unphysical representations of the physical system of interest: using fairly coarse spatial resolution of the fluid, lasting for only tens of rotation periods (compared to tens of thousands needed in the physical case), and using unphysical strengths for the GR reaction force (absent completely in Ref. [43], and thousands of times stronger than the physical case in Refs. [44, 45]). Recent analytical studies by Schenk, *et. al* [46], Morsink [47], and Arras, *et. al* [48] were able to overcome these limitations. They studied the non-linear hydrodynamic effects on the growth of an unstable r-mode by considering non-linear coupling between modes in the weak coupling limit. They show that an unstable r-mode couples strongly to other modes in this way. When the amplitude of an unstable r-mode reaches a certain level, α_{sat}, the energy which drives the instability is diverted by these non-linear couplings into a cascade which excites hundreds (or thousands) of other modes where the energy is ultimately converted to heat through viscous dissipation. Arras, *et. al* [48] compute the level where this saturation of the GR driven r-mode instability occurs to be

$$\alpha_{sat} \approx 0.005\sqrt{\frac{\alpha_e}{0.1}}\sqrt{\frac{2\times 10^5}{\tau_{GR}\Omega_{\max}}}\left(\frac{\Omega}{\Omega_{\max}}\right)^{5/2}, \tag{14}$$

where $\tau_{GR}\Omega_{\max}$ is the dimensionless product of the GR growth time and the angular velocity of the most rapidly rotating star. The "matching parameter" α_e is a measure of the strength of the mode-mode coupling, which is estimated to be in the range $4\times 10^{-4} < \alpha_e < 0.1$. So non-linear hydrodynamic forces will limit the growth of the r-mode amplitude to values in the range: $10^{-4} < \alpha_{sat} < 5\times 10^{-3}$. Such small amplitudes will prevent the r-modes from radiating significant amounts of GR before the neutron star becomes quite cold and consequently, according to our present understanding, quite stable. This limitation on the amplitude of the r-mode also limits the flux of GR that could be emitted during any period of instability to levels that are unlikely to be detectable.

4 Concluding Remarks

At the present time it appears that the GR instability in the r-modes may be not be strong enough to overcome the numerous dissipative processes that act to suppress it in real neutron stars. But there remain a number of important questions effecting this conclusion which have yet to be completely resolved. It is not yet completely understood (although see Refs. [36, 37, 30, 49, 50]) what role magnetic fields play in the evolution of the r-modes. Will magnetic fields suppress the instability, limit its growth, or merely change the values of the frequency and growth times? Is the formation of a solid crust delayed long enough by differential rotation or pulsations after the birth of a neutron star to allow the r-mode instability to act? Do semi-rigid crust effects move the critical angular velocity to small enough values that the GR instability can act in the LMXBs? Do superfluid effects (*e.g.*, pinning of the core vortices or vortex-fluxtube dissipation) suppress the r-mode instability completely in these stars? Does the equation of state of real neutron-star matter contain hyperons, kaons, or even free quarks which strongly increase the dissipation in the r-modes? And do rapid direct-URCA type interactions cool young stars on timescales that are short compared to the r-mode instability timescale?

Acknowledgments

It is a pleasure to thank Y. Eriguchi and M. Shibata for arranging my visit to Tokyo and for making my stay so pleasant. I also thank my collaborators N. Andersson, C. Cutler, J. Ipser, G. Mendell, S. Morsink, B. Schutz, J. Tohline, G. Ushomirsky, M. Vallisneri, A. Vecchio and especially B. Owen for their help in working out most of the material on r-modes presented in this paper. This work was supported by NASA grant NAG5-10707 and NSF grant PHY-0099568.

References

[1] L. Lindblom, in *Gravitational Waves: A challenge to Theoretical Astrophysics*, ICTP Lecture Notes Series Volume III, edited by V. Ferrari, J.C. Miller, & L. Rezzolla (2001) pp. 257–275.

[2] K.S. Thorne, and A. Campolattaro, Astrophys. J. **149**, 591 (1967); **152**, 673 (1968).

[3] K.S. Thorne, Astrophys. J. **158**, 1 (1969).

[4] S. Chandrasekhar, Phys. Rev. Letters **24**, 611 (1970).

[5] S. Chandrasekhar, Astrophys. J. **161**, 561 (1970).

[6] J.L. Friedman, and B.F. Schutz, Astrophys. J. **222**, 281 (1978).

[7] J.L. Friedman, Comm. Math. Phys. **62**, 247 (1978).

[8] L. Lindblom, and S.L. Detweiler, Astrophys. J. **211**, 565 (1977).

[9] L. Lindblom, and W.A. Hiscock, Astrophys. J. **267**, 384 (1983).

[10] C. Cutler, L. Lindblom, and R.J Splinter, Astrophys. J. **363**, 603, (1990).

[11] J.R. Ipser, and L. Lindblom, Astrophys. J. **373**, 213 (1991).

[12] C. Cutler, and L. Lindblom, Astrophys. J. **385**, 630 (1992).

[13] L. Lindblom, Astrophys. J. **438**, 265 (1995).

[14] N. Andersson, Astrophys. J. **502**, 708 (1998).

[15] J.L. Friedman, and S.M. Morsink, Astrophys. J. **502**, 714 (1998).

[16] L. Lindblom, B.J. Owen, and S.M. Morsink, Phys. Rev. Letters **80**, 4843 (1998).

[17] B.J. Owen, L. Lindblom, C. Cutler, B.F. Schutz, A. Vecchio, and N. Andersson, Phys. Rev. D **58**, 084020 (1998).

[18] J. Papaloizou, and J.E. Pringle, Mon. Not. R. Astron. Soc. **182**, 423 (1978).

[19] L. Lindblom, G. Mendell, and B.J. Owen, Phys. Rev. D **60**, 064006 (1999).

[20] B.J. Owen, and L. Lindblom, Class. Quant. Grav. **19** 1247 (2002).

[21] L. Bildsten, Astrophys. J. **501**, L89 (1998).

[22] N. Andersson, K.D. Kokkotas, and N. Stergioulas, Astrophys. J. **516**, 307 (1999).

[23] R.V. Wagoner, Astrophys. J. **278**, 345 (1984).

[24] Y. Levin, Astrophys. J. **517**, 328 (1999).

[25] N. Andersson, D.I. Jones, K.D. Kokkotas, and N. Stergioulas, Astrophys. J. **534**, L75 (2000).

[26] J.L. Zdunik, P. Haensel, B. Paczyńzki, and J. Miralda-Escudé, Astrophys. J. **384**, 129 (1992).

[27] E.F. Brown, Astrophys. J. **531**, 988 (2000).

[28] L. Lindblom, and G. Mendell, Phys. Rev. D **61**, 104003 (2000).

[29] L. Bildsten, and G. Ushomirsky, Astrophys. J. **529**, L33 (2000).

[30] G. Mendell, Phys. Rev. D **64**, 044009 (2001).

[31] C.J. Pethick, and V. Thorsson, in *Lives of the Neutron Stars*, ed. by a. Alpar, Ü. Kiziloglu, and J. van Paradijs (Kluwer, Dordrecht: 1995).

[32] D. Page, M. Prakash, J. M. Lattimer, and A. Steiner, Phys. Rev. Lett. **85**, 2048 (2000).

[33] M. Rieutord, Astrophys. J. **550** 443 (2001); Astrophys. J. **557** 493 (2001).

[34] L. Lindblom, B.J. Owen, and G. Ushomirsky, Phys. Rev. D **62**, 084030 (2000).

[35] Y. Levin, and G. Ushomirsky, Mon. Not. Roy. Astro. Soc. **324**, 917 (2001).

[36] L. Rezzolla, F. K. Lamb, and S.L. Shapiro, Astrophys. J. **531**, L139 (2000).

[37] W.C.G. Ho, and D. Lai, Astrophys. J. **543**, 386 (2000).

[38] P. Slane, D.J. Helfand, S.S.Murray, Astrophys. J. (in press) (2002); astro-ph/0204151.

[39] Y. Wu, C.D. Matzner, and P. Arras, Astrophys. J. **549**, 1011 (2001).

[40] P. Jones, Phys. Rev. Lett. **86**, 1384 (2001).

[41] P. Jones, Phys. Rev. D **64**, 084003 (2001).

[42] L. Lindblom, and B.J. Owen, Phys. Rev. D **65**, 063006 (2002).

[43] N. Stergioulas, and J.A. Font, Phys. Rev. Lett., **86** 1148 (2001).

[44] L. Lindblom, J.E. Tohline, and M. Vallisneri, Phys. Rev. Lett. **86** 1152 (2001).

[45] L. Lindblom, J.E. Tohline, and M. Vallisneri, Phys. Rev. D **65** 084039 (2002).

[46] A.K. Schenk, P. Arras, E.E. Flanagan, S.A. Teukolsky, I. Wasserman, Phys. Rev. D **65**, 024001 (2002).

[47] S.M. Morsink, astro-ph/0202051.

[48] P. Arras, E.E. Flanagan, S.M. Morsink, A.K. Schenk, S.A. Teukolsky, I. Wasserman, astro-ph/0202345.

[49] L. Rezzolla, F. L. Lamb, D. Markovic, and S. L. Shapiro, Phys. Rev. D **64**, 104013 (2001).

[50] L. Rezzolla, F. L. Lamb, D. Markovic, and S. L. Shapiro, Phys. Rev. D **64**, 104014 (2001).