THE EFFECTS OF SUPERFLUID HYDRODYNAMICS ON THE STABILITY OF ROTATING NEUTRON STARS

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Rotating stars are subject to a secular instability driven by the emission of gravitational radiation.[1] This instability is generic: all rotating stars composed of perfect fluid are unstable in general relativity theory.[2,3] However, sufficiently slowly rotating stars composed of a viscous fluid are stable.[4,5] The calculation then of exactly which rotating neutron stars are stable is delicate. Such a calculation must determine the relative strengths of the destabilizing gravitational radiation and the stabilizing viscosity effects on the relevant modes of these stars.[6,7]

Below about 10^9K neutron star matter is expected to undergo a phase transition into a superfluid state.[8,9] In this state the protons and neutrons in the core of the neutron star are expected to behave as a complicated superconducting-superfluid mixture. The dynamical and dissipative properties of this mixture are expected to be significantly different from those of a perfect fluid. Thus it is necessary to consider anew the secular stability of rotating neutron stars using superfluid hydrodynamics. We present here new estimates of the effects of the superfluid dissipation mechanism "mutual friction" on the secular stability of rotating neutron stars.

The oscillations of a rotating superfluid neutron star can be described by just three scalar potentials: δU, $\delta\beta$, and $\delta\Phi$.[10] The scalar $\delta\Phi$ is the Newtonian gravitational potential, while the remaining potentials describe the thermodynamic state of the fluid: $\delta U \equiv \delta p/\rho + \delta\Phi$ is related to the pressure perturbation while $\delta\beta$ is the deviation of the fluid state from β-equilibrium. These scalars are determined by three coupled second-order (typically elliptic) differential equations plus appropriate boundary conditions. Here we use simple analytical solutions to these equations that describe the oscillations of spatially homogeneous non-rotating neutron star matter.[10] In this simple case the potentials are separable: the radial dependences are suitable linear combinations of r^l and the spherical Bessel functions j_l while the angular dependences are the spherical harmonics Y_{lm}.

The effects of dissipation on these oscillations are conveniently analyzed in terms of an energy functional E: a real integral of a quadratic form in the three potentials δU, $\delta\beta$, and $\delta\Phi$. A suitable energy functional has been found for this superfluid hydrodynamic system.[11] Its time derivative (which vanishes in the absense of dissipation) is given by another functional, dE/dt, that describes the effects of gravitational radiation, viscosity, and the purely superfluid dissipative effect "mutual friction." Mutual friction in superfluid neutron star matter is caused by the scattering of electrons off the cores of the superfluid vortices. Using these functionals it is possible to evaluate the imaginary part of the frequency of any mode: $\mathrm{Im}(\omega) = (1/2E)(dE/dt) \equiv -1/\tau$. The contributions of the dissipative effects to $1/\tau$ are denoted: $1/\tau_{GR}$ for gravitational radiation, $1/\tau_V$ for viscosity, and $1/\tau_{MF} \equiv (\Omega/\Omega_o)(1/\tilde{\tau}_{MF})$ for mutual friction, where Ω is the angular velocity of the rotating star and $\Omega_o \equiv \sqrt{\pi G\rho}$. We use our simple analytical expressions for δU, $\delta\beta$, and $\delta\Phi$ to estimate these damping times for the relevant modes. Table 1 gives these quantities for the $2 \le l = m \le 6$ modes for a uniform density model with $\rho = 4 \times 10^{14}$ gm/cm^3 and a radius of $R = 15$ km.

Table 1. Fundamental Frequencies and Damping Times

l	$\frac{\omega}{\Omega_o}$	$\tau_{GR}\Omega_o$	$\tilde{\tau}_{MF}\Omega_o$	$\tau_V\Omega_o$	$\left\lvert\frac{\delta\beta}{\delta U}\right\rvert$
2	1.406	2.680×10^3	1.480×10^4	7.222×10^{11}	0.115
3	1.808	1.301×10^5	2.290×10^4	2.852×10^{11}	0.089
4	2.141	5.699×10^6	3.319×10^4	1.538×10^{11}	0.072
5	2.430	2.609×10^8	4.561×10^4	9.621×10^{10}	0.060
6	2.689	1.289×10^{10}	6.017×10^4	6.581×10^{10}	0.052

We wish to estimate the critical values of the angular velocity of a star where the imaginary part of the frequency changes sign from negative (stable) to positive (unstable): $1/\tau(\Omega_c) = 0$. In analogy with the analysis of perfect fluid stars[7,12] these critical angular velocites may be estimated by solving the equation

$$\frac{\Omega_c}{\Omega_o} \approx \frac{1}{l}\frac{\omega}{\Omega_o}\left[1 + \left(\frac{\tau_{GR}}{\tau_V} + \frac{\Omega_c}{\Omega_o}\frac{\tau_{GR}}{\tilde{\tau}_{MF}}\right)^{1/(2l+1)}\right]. \tag{1}$$

We have solved this equation numerically for the $2 \le l \le 6$ modes using the frequencies and damping times from Table 1. We find that the smallest solution (the $l = 2$ mode) is $\Omega_c \approx 1.2\Omega_o$. This is unphysical since it is larger than the maximum value of the angular velocity $\Omega_{MAX} \approx 0.7\Omega_o$ for which there exists an equilibrium stellar model. Since the magnitude of $|\delta\beta|$ is larger in these simple solutions than in more realistic ones[10] (by a factor of about 10^2) we expect that the magnitude of $\tilde{\tau}_{MF}$ given here may be too small (by a factor of about 10^4) and hence that the supression of the gravitational radiation instability is too large. We also solved Eq. (1) using values of $\tilde{\tau}_{MF}$ which are 10^6 times the values given Table 1. Even in this case the smallest critical angular velocity (in the $l = 3$ mode in this case) exceeds $0.7\Omega_o$. Thus, we conclude that mutual friction will suppress the gravitational radiation driven secular instability in all rotating neutron stars cooler than the superfluid transition temperature.

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