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Superfluid Effects on the Stability of Rotating Neutron Stars

Gravitational radiation tends to make all rotating stars unstable, while internal dissipation (e.g., viscosity) tends to counteract this instability. The purpose of this paper is to investigate how the internal dissipation mechanisms that exist within superfluid neutron-star matter influence the gravitational-radiation instability. We argue here that mutual friction between the electrons and the quantized neutron vortices completely suppresses the gravitational-radiation instability in rotating neutron stars cooler than the superfluid-transition temperature.

1. DISSIPATION IN ROTATING SUPERFLUIDS

Below the superfluid-transition temperature ($T \approx 10^9$ K) the dissipation mechanisms are significantly different from those in warmer neutron-star matter. The neutrons and protons form Cooper pairs and condense into superfluid states. Scattering among these particles is prevented by the energy gap that separates these states from the normal-particle states. Thus scattering is confined primarily to the electrons in superfluid neutron stars. These may scatter with each other to dissipate energy via ordinary shear viscosity. In rotating neutron stars another dissipation mechanism exists: mutual friction—the scattering of normal particles off the quantized vortices of the superfluid condensates.[2] In the case of a neutron-star superfluid the scattering of electrons with the vortices of the neutron superfluid dominates this dissipation. Due to Fermi-liquid "drag" effects the neutron vortices carry a substantial magnetic flux with which the electrons may scatter.[1] The hydrodynamic equations (including dissipation) needed to analyze in detail the large-scale dynamics of rotating superfluid neutron-star matter have been developed recently by Mendell.[4] We use some of the results of that work to estimate here the effectiveness of mutual friction in suppressing the gravitational-radiation instability in rotating neutron stars.

2. INSTABILITIES IN SUPERFLUID NEUTRON STARS

Consider the perturbations of a neutron star having time dependence $e^{i\omega t - t/\tau}$, where ω and τ are real. Such a perturbation is stable whenever $1/\tau \geq 0$. Thus in a sequence of rotating stars parameterized by Ω (the angular velocity), those stars rotating slower than the smallest root of $1/\tau(\Omega_c) = 0$ are stable. (Assuming of course that the star having $\Omega = 0$ is stable.)

In superfluid neutron stars the important dissipative damping times are τ_{η_e} due to shear viscosity, τ_{MF} due to mutual friction, and τ_{GR} due to gravitational radiation. These determine the total damping time by $1/\tau = 1/\tau_{\eta_e} + 1/\tau_{MF} + 1/\tau_{GR}$. The viscous and gravitational-radiation damping times are discussed in some detail in Lindblom's review in this volume. That discussion will not be repeated here. The mutual-friction damping time τ_{MF} may be expressed as an integral involving the perturbations:[4]

$$\frac{1}{\tau_{MF}} = \frac{\Omega}{2E} \int \beta_{MF} \left[\Delta\vec{v} \cdot \Delta\vec{v}^{\,*} - |\Delta\vec{v} \cdot \vec{z}\,|^2 \right] d^3x, \tag{1}$$

where E is an energy of the perturbations, β_{MF} is the strength of the mutual-friction interaction, $\vec{z}$ is the unit vector parallel to the rotation axis, and $\Delta\vec{v}$ is the perturbation of the relative velocity between the neutron superfluid and the charged fluids. (The charged fluids are coupled electromagnetically on timescales much shorter than τ_{MF}.[1]) An estimate for β_{MF} has been given by Mendell[4] based on the calculation of electron-vortex scattering by Alpar, Langer, and Sauls:[1]

$$\beta_{MF} = 1.0 \times 10^{-4} \rho_p^{7/6} \left(1 - \frac{m_n}{m_n^*} - \frac{m_p}{m_p^*}\right)^2 \left(1 - \frac{m_p^*}{m_p}\right)^2 \left(\frac{m_p}{m_p^*}\right)^{1/2}, \tag{2}$$

where m_n/m_n^* and m_p/m_p^* are the ratios of the masses to the effective masses of the neutrons and protons respectively, and ρ_p is the proton mass density.

The mutual-friction damping times have yet to be evaluated for the pulsations of rotating neutron stars. The magnitude of τ_{MF} may be estimated, however, from Eq. (1) for the plane-wave perturbations of uniform-density neutron-star matter.[4] For perturbations with wavelength $\lambda = 2\pi R/l$, the ratio of the mutual friction to the viscous damping time is found to be

$$\frac{\tau_{MF}}{\tau_{\eta_*}} < 10^{-6}\, l^2 \left(\frac{10^6\ \mathrm{cm}}{R}\right)^2 \left(\frac{10^3\ \mathrm{s}^{-1}}{\Omega}\right) \left(\frac{10^9\ \mathrm{K}}{T}\right)^2, \tag{3}$$

This limit on the mutual-friction damping time can be used to place limits on the roots of $1/\tau(\Omega_c) = 0$. We use the angular-velocity dependence of τ_{η_*} and τ_{GR} computed by Ipser and Lindblom[3] for the $2 \le l = m \le 6$ f-modes of rotating neutron stars. We find *no* roots of $1/\tau(\Omega_c) = 1/\tau_{\eta_*}(\Omega_c) + 1/\tau_{MF}(\Omega_c) + 1/\tau_{GR}(\Omega_c) = 0$ for these modes if the temperature is less than 10^9 K. Thus, we conclude that mutual friction eliminates the gravitational-radiation instability in all rotating neutron stars cooler than the superfluid transition temperature.

ACKNOWLEDGMENT

This research was supported by NSF grant PHY-9019753.

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