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Magnetic field amplification by the r-mode instability

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Abstract. We discuss the magnetic field enhancement by unstable r-modes (driven by the gravitational radiation reaction force) in rotating stars. In the absence of a magnetic field, gravitational radiation exponentially increases the r-mode amplitude α, and accelerates differential rotation (secular motion of fluid elements). For a magnetized star, differential rotation enhances the magnetic field energy. Rezzolla et al (2000–2001) argued that if the magnetic energy grows faster than the gravitational radiation reaction force pumps energy into the r-modes, then the r-mode instability is suppressed. Chugunov (2015) demonstrated that without gravitational radiation, differential rotation can be treated as a degree of freedom decoupled from the r-modes and controlled by the back reaction of the magnetic field. In particular, the magnetic field windup does not damp r-modes. Here we discuss the effect of the back reaction of the magnetic field on differential rotation of unstable r-modes, and show that it limits the generated magnetic field and the magnetic energy growth rate preventing suppression of the r-mode instability by magnetic windup at low saturation amplitudes, $\alpha \ll 1$, predicted by current models.

1. Introduction

Neutron stars contain matter which is denser than that in atomic nuclei. Such a matter cannot be produced in terrestrial laboratories. Accordingly, neutron stars provide a unique opportunity to study the properties of super-dense matter by comparing observations of these stars with theoretical models. One such possibility is associated with the gravitational radiation driven instability of r-modes in rapidly rotating neutron stars [1–7]. In the absence of dissipation and magnetic fields, the r-modes are unstable in any (even slowly) rotating star [6, 7]. Dissipation (e.g., shear and bulk viscosities [8, 9], an Ekman layer [10, 11], mutual friction [12–16]) can suppress this instability to some extent. However if a neutron star spin frequency exceeds a (temperature dependent) critical value, it can be unstable (see e.g., [17] for recent review), and the instability will alter the star's evolution [5, 18–20]. Observations of rapidly rotating neutron stars (i.e., neutron stars in accreting low mass X-ray binaries and their descendants

– millisecond pulsars) put important constraints on the r-mode instability threshold and thus on properties of super-dense matter (c.f. [21–26]). However, these constraints assume that the internal fluid dissipation is the main damping mechanism for the r-modes; such constraints can be irrelevant if the r-mode instability is, in fact, suppressed by an alternate mechanism like a magnetic field windup [27–29]. These publications argue that the growth of unstable r-modes should be accompanied by differential rotation and associated bending of the magnetic field lines powered by the r-mode energy. If the r-mode energy loss exceeds the energy supplied by the gravitational radiation instability, the r-modes should be damped out. Here we summarize our analysis of the back reaction of bent magnetic lines on the differential rotation, and obtain an upper limit on the the magnetic field enhancement by the r-mode instability. We show that magnetic windup cannot suppress the r-mode instability if the nonlinear saturation amplitude is low, as predicted by current theoretical models, e.g. [30–32]. More details can be found in [33].

2. R-modes and differential rotation

Differential rotation which suppresses the gravitational radiation driven instability in the magnetic windup scenario [27–29], is a second-order effect in the r-mode amplitude. Before considering the full problem (including the effects of gravitational radiation and magnetic fields), we outline previously published results on two simplified problems: (1) the r-modes in non-magnetized stars and (2) stable r-modes (i.e. neglecting gravitational radiation) in magnetized stars.

The solution for stable r-modes in non-magnetized stars to second-order in amplitude α was obtained in [34]. The axisymmetric part of this solution describes differential rotation and contains a gauge dependent part; the general solution includes contribution, corresponding to the second order differential rotation allowed in non-oscilating star (i.e., an arbitrary time independent azimuthal flow, stratified on cylinders). A particular solution with vanishing drift velocity (no secular motion of fluid elements) exists. However, the latter solution does not mean that the axisymmetric oscillation averaged second order velocity perturbations are absent. On the contrary, according to [34] these terms are not zero and are not cylindrically stratified for any choice of initial conditions. The apparent contradiction between the absence of secular motion of the fluid elements and non-vanishing velocity perturbations is superficial. It is associated with Stokes drift [36, 37] – second order secular motion of fluid elements associated with the oscillating velocity profile in first order of perturbation theory. The Stokes drift contributes to the secular motion along with the oscillation averaged velocity perturbations and resolves the contradiction (see [35] for details).

Recently, an analytical second order solution for the unstable r-modes in non-magnetized rotating stars was obtained [38], neglecting internal fluid dissipation. This solution demonstrates an exponential growth of the r-mode amplitude, $\alpha(t) = \alpha(0)\exp(t/\tau_{\rm GR})$ (with the time scale $\tau_{\rm GR} > 0$ determined previously in [8]). The growth is accompanied by the exponential growth of differential rotation (with a particular spatial profile determined by the equation of state). Like a stable r-mode, this solution has a gauge freedom – the general solution includes an arbitrary second order time independent differential rotation profile. The secular motion of fluid elements is still cylindrically stratified, but the solution with vanishing drift is absent; the exponential growth of velocity perturbations leads to an exponential growth of fluid element displacements.

For a magnetized star with perfect conductivity, the secular motion of fluid elements leads to bending of magnetic field lines. A solution with time independent non-vanishing drift velocity becomes impossible. This was confirmed in [35] where the general second order solution for the stable r-modes in a magnetized neutron star was obtained. As with non-magnetized stars, the general solution has gauge freedom; it is associated with perturbations of a non-oscillating star and can be described as an ensemble of Alfvén modes. A solution with the vanishing drift of fluid elements exists for specific initial conditions. The general second order solution can be presented as a superposition of two solutions: (a) a solution which describes the evolution of differential rotation in a non-oscillating magnetized star (i.e. an ensemble of Alfvén modes, determined by initial conditions) and (b) the r-mode solution with vanishing drift. This solution demonstrates that stable r-modes are not damped by magnetic windup.

The criteria for the gravitation radiation driven instability was generalized to magnetized stars in [39]. However this result cannot be directly applied to check the magnetic windup scenario. This scenario assumes the r-mode instability is not suppressed initially, but it can be suppressed by the growth of the unstable r-mode leading to windup of the magnetic field and subsequent suppression of the instability.

The magnetic field evolution in the presence of an unstable r-mode is analyzed in our recent paper [33] using second order perturbation theory for an infinitely conducting rotating star including gravitational radiation reaction forces. The results are illustrated there by a simple toy model, and confirmed by a detailed analysis of the full problem using the symplectic product formalism [2, 40]. They can be summarized as follows. In second order perturbation theory, the gravitational radiation reaction and magnetic forces have axisymmetric components with non-vanishing oscillation averages that directly affect the secular motion of fluid elements. These components are (a) a gravitational radiation reaction force, with magnitude (per unit mass) $f_{\rm GR} \sim \alpha^2(t)\,\tau_{\rm GR}^{-1}\,\Omega R$, where $\Omega = 2\pi\nu$ is the spin frequency of the star, and (b) a force associated with the second order magnetic terms in the perturbation equations, with magnitude (per unit mass) $f_{\rm m} \sim \alpha^2(t)\,\omega_{\rm A}\Omega$ [33], assuming a non-superconducting stellar interior. Here, $\omega_{\rm A} = \sqrt{\pi B^2/\rho R^2}$ is a typical frequency of Alfvén modes, B is an initial magnetic field, ρ is a typical density, and R is the stellar radius. Before saturation, when the mode amplitude $\alpha(t)$ is exponentially growing, $\alpha(t) = \alpha(0)\exp(t/\tau_{\rm GR})$, the displacements of fluid elements in the azimuthal direction can be estimated in the same way as the displacement of an “Alfvén” harmonic oscillator with frequency $\omega_{\rm A}$ evolving under the action of an exponentially growing external force $f_{\rm GR} + f_{\rm m}$ (with timescale $\tau_{\rm GR}/2$). The result is [33]

$$\xi^{\hat{\phi}} \sim \frac{f_{\rm GR} + f_{\rm m}}{\left(2\tau_{\rm GR}^{-1}\right)^2 + \omega_{\rm A}^2} \sim \alpha^2(t)\,\Omega R\,\frac{\max(\omega_{\rm A},\,\tau_{\rm GR}^{-1})}{4\tau_{\rm GR}^{-2} + \omega_{\rm A}^2} \leq \alpha^2(t)\,R\,\frac{\Omega}{\omega_{\rm A}}, \tag{1}$$

where the last inequality follows from a detailed analysis of the terms. This estimate also allows us to estimate the magnetic field energy density,

$$\epsilon_{\rm m} \sim \rho\,\omega_{\rm A}^2 \left(\xi^{\hat{\phi}}\right)^2 \leq \alpha^4(t)\,\rho\,\Omega^2 R^2 \tag{2}$$

and its growth rate $\dot{\epsilon}_{\rm m} \sim 4\alpha^4(t)\,\tau_{\rm GR}^{-1}\,\rho\,\Omega^2 R^2$, again using the harmonic oscillator analogy, or a more detailed mathematical analysis of the full problem [33]. The energy density of r-modes can be estimated as $\epsilon_{\rm r} \sim \alpha^2(t)\,\rho\Omega^2 R^2$ (c.f. [8]), and its rate as $\dot{\epsilon}_{\rm r} \sim 2\alpha^2(t)\,\tau_{\rm GR}^{-1}\,\rho\,\Omega^2 R^2 \sim \alpha^{-2}(t)\dot{\epsilon}_{\rm m} \gg \dot{\epsilon}_{\rm m}$. Therefore, the magnetic wind up cannot suppress the r-mode instability before saturation (unless the saturation amplitude $\alpha(t) \sim 1$, as assumed in early papers [27–29]).

In the simplest model of nonlinear r-mode saturation, the amplitude growth simply stops and remains frozen once $\alpha(t) = \alpha_{\rm sat}$. In this case, $\xi^{\hat{\phi}}$ can be estimated as the displacement of a harmonic Alfvén oscillator under the influence of the constant force $f_{\rm GR} + f_{\rm m}$ with mode amplitude $\alpha = \alpha_{\rm sat}$,

$$\xi^{\hat{\phi}}_{\rm sat} \sim \frac{f_{\rm GR} + f_{\rm m}}{\omega_{\rm A}^2} \sim \alpha_{\rm sat}^2\,\Omega R\,\frac{\max(\omega_{\rm A},\,\tau_{\rm GR}^{-1})}{\omega_{\rm A}^2}. \tag{3}$$

It can significantly exceed the displacement in the exponential growth phase, $\xi^{\hat{\phi}}$, if $\omega_{\rm A} \ll \tau_{\rm GR}^{-1}$. The growth rate of the magnetic energy can be estimated as $\dot{\epsilon}_{\rm m} \sim \omega_{\rm A}\epsilon_{\rm m}$, with $\epsilon_{\rm m}$ given by

(2) with $\xi^{\hat{\phi}} = \xi^{\hat{\phi}}_{\rm sat}$. Formally, $\dot{\epsilon}_{\rm m}$ is comparable to $\dot{\epsilon}_{\rm r}$ if $\omega_{\rm A} \sim \alpha^2_{\rm sat}\tau^{-1}_{\rm GR} \ll \tau^{-1}_{\rm GR}$. However, even for an unexpectedly large $\alpha_{\rm sat} \sim 10^{-3}$ in a non-superconducting neutron star, this will affect the r-mode instability only if the initial magnetic field is $\lesssim 100\,{\rm G}$. Alternatively, the magnetic windup cannot suppress the r-mode instability. Equation (3) can also be used to estimate the maximum azimuthal magnetic field, generated by a magnetic windup in a non-superconducting star,

$$\delta B^{\hat{\phi}} \sim B\,\frac{\xi^{\hat{\phi}}}{R} \lesssim 4\times 10^{8}\ \left(\frac{\alpha}{10^{-4}}\right)^2 \frac{\max(\omega_{\rm A},\,\tau^{-1}_{\rm GR})}{\omega_{\rm A}}\,\frac{\nu}{500\,{\rm Hz}}\,\frac{R}{10^6\,{\rm cm}}\,\frac{\rho}{4\times 10^{14}\,{\rm g\,cm^{-4}}}\,{\rm G}. \qquad (4)$$

3. Summary and conclusions

We have outlined nonlinear (second order in perturbation theory) unstable r-modes in magnetized stars composed of a perfect fluid with infinite conductivity. Previously, the second order r-modes were studied for non-magnetized and magnetized Newtonian stars ([34] and [35], respectively), and for non-magnetized stars with gravitational radiation reaction [38]. Our approach here includes both gravitational radiation reaction and magnetic forces and, therefore, provides a self-consistent description of the r-mode instability in magnetized stars. This problem is important because of the magnetic windup scenario [27–29] in which the r-mode instability leads to magnetic field enhancement that suppresses the r-mode instability. Our results confirm the suggestion [27–29], that the growing r-modes induce the differential drift of fluid elements and generation of magnetic fields. However, before nonlinear saturation is achieved [i.e., at the stage of exponentially growing $\alpha(t)$], the magnetic energy growth rate is a factor of $\alpha^2(t)$ smaller than the r-mode energy growth rate by radiation reaction. Thus magnetic windup cannot affect the r-mode instability if $\alpha(t) \ll 1$. After the saturation, the enhancement of the magnetic energy is also strongly restricted. Current models of nonlinear saturation [30–32] suggest that the r-mode amplitude is limited by $\alpha \sim \alpha_{\rm sat} \lesssim 10^{-4}$. Therefore, the magnetic windup cannot suppress the instability in this case. We have also estimated the maximal magnetic field that could be generated by the r-mode instability. For an unexpectedly large saturation amplitude $\alpha_{\rm sat} = 10^{-3}$, an initial magnetic field 10^8 G could be amplified up to about 10^{11} G, which can be important for the magnetic field generation in neutron stars. However if the initial magnetic field is $\sim 10^{10}$ G, it would not be significantly affected by the r-mode magnetic windup. The assumption that the magneto-rotational instability does not operate during magnetic windup should be carefully checked in the future.

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