

Gravitational helioseismology?

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The magnitudes of the external gravitational perturbations associated with the normal modes of the Sun are evaluated to determine whether these solar oscillations could be observed with the proposed laser interferometer space antenna (LISA), a network of satellites designed to detect gravitational radiation. The modes of relevance to LISA—the $l=2$, low-order p , f , and g modes—have not been conclusively observed to date. We find that the energy in these modes must be greater than about 10^{30} ergs in order to be observable above the LISA detector noise. These mode energies are larger than generally expected, but are much smaller than the current observational upper limits. LISA may be confusion limited at the relevant frequencies due to the galactic background from short-period white dwarf binaries. Present estimates of the number of these binaries would require the solar modes to have energies above about 10^{33} ergs to be observable by LISA. [S0556-2821(96)04914-4]

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I. INTRODUCTION

The oscillation modes of the Sun are now routinely observed by Doppler-shift measurements of the time-dependent velocity of the Sun's surface [1,2]. The frequencies and amplitudes of thousands of p modes have now been accurately measured in this way. A number of detections of solar g modes from Doppler-shift measurements have also been reported, but the claimed detections all have low signal-to-noise and they are not mutually consistent [3]. It has recently been claimed (although, again, it is not universally accepted) that certain periodic features observed in the solar wind provide a second, completely independent, means of observing the solar oscillations [4]. Some of the frequencies observed in the solar wind correspond closely to already observed solar p modes, while others are close to frequencies predicted for some solar g modes.

In this paper we investigate the possibility (also suggested independently by Schutz [5] and by Gough [6]) that the solar oscillations might be observable using a third technique: measuring the external gravitational perturbations associated with these modes. The proposed laser interferometer space antenna (LISA) [7] is designed to detect the tiny gravitational fluctuations caused by distant sources of gravitational radiation with frequencies in the range $10^2 - 10^6$ μHz . This detector would also be capable of observing the fluctuations of the near-zone gravitational field produced by the normal modes of the Sun, if the amplitudes of the solar oscillations are sufficiently large. This new observational technique would be an interesting new probe of the structure of the Sun because it would be most sensitive for detecting a class of low-order modes that are presently unobserved using other methods. In this paper we calculate the magnitudes of the external gravitational perturbations associated with the solar p , f , and g modes to estimate whether these oscillations could be observed with LISA.

II. SOLAR GRAVITATIONAL PERTURBATIONS

We evaluate the coupling of the pulsation modes of the Sun to the external gravitational field using a reasonably accurate model for the structure of the Sun and its dynamics. We use the equation of state of a normal solar model (model 1 of Christensen-Dalsgaard, Proffitt, and Thompson [8]) to determine the equilibrium structure of the Sun. We treat the pulsations as small amplitude perturbations which satisfy linear adiabatic evolution equations [9] with a realistic dynamical adiabatic index [8]. We solve the equations for these perturbations numerically to determine the magnitude of the external gravitational perturbation that results from a mode of given amplitude. The gravitational perturbation in the exterior of the Sun has the form

$$\delta\Phi = \alpha_{28} \left(\frac{E}{10^{28} \text{ ergs}} \right)^{1/2} \frac{GM_{\odot}}{R_{\odot}} \left(\frac{R_{\odot}}{r} \right)^{l+1} Y_{lm} e^{i\omega t}, \quad (1)$$

where r is the distance from the center of the Sun, and Y_{lm} is the standard spherical harmonic function. The factor α_{28} represents the magnitude of this external gravitational perturbation for an oscillation mode normalized to have energy $E = 10^{28}$ ergs. The energy E is defined as

$$E = \int \rho \delta v_a^* \delta v^a d^3x, \quad (2)$$

where ρ is the mass density and δv^a is the (Eulerian) fluid velocity perturbation. The values of α_{28} that we compute for a number of solar oscillation modes are presented in Table I. The magnitude of the external gravitational perturbation of a mode scales as \sqrt{E} since E is quadratic in the perturbed fluid velocity. The values of α_{28} given in Table I illustrate that the largest surface gravitational perturbation occurs in the $l=2$ g_3 mode when modes excited to equal energy levels are compared. Also presented in Table I are the mode masses,

TABLE I. Gravitational parameters of solar oscillations.

Mode	f (μHz)	α_{28}	M_{mode} (g)	h	E_{min} (erg)	E_{max} (erg)
g_3	220.4	7.17×10^{-12}	8.3×10^{31}	1.18×10^{-23}	1.55×10^{30}	1.3×10^{33}
g_2	254.0	6.07×10^{-12}	4.4×10^{31}	7.50×10^{-24}	1.97×10^{30}	7.0×10^{32}
p_1	381.6	5.31×10^{-12}	4.3×10^{30}	2.91×10^{-24}	2.02×10^{30}	6.9×10^{31}
g_4	192.2	5.96×10^{-12}	1.9×10^{32}	1.29×10^{-23}	2.42×10^{30}	3.0×10^{33}
g_5	168.8	4.72×10^{-12}	3.5×10^{32}	1.32×10^{-23}	4.19×10^{30}	5.6×10^{33}
g_1	293.6	2.81×10^{-12}	2.3×10^{31}	2.60×10^{-24}	8.44×10^{30}	3.7×10^{32}
f	350.9	2.31×10^{-12}	1.2×10^{31}	1.50×10^{-24}	1.12×10^{31}	1.9×10^{32}
p_2	514.4	2.02×10^{-12}	5.5×10^{29}	6.07×10^{-25}	1.17×10^{31}	8.8×10^{30}
g_6	134.0	2.80×10^{-12}	8.2×10^{32}	1.24×10^{-23}	1.37×10^{31}	1.3×10^{34}
p_3	663.6	8.60×10^{-13}	1.4×10^{29}	1.56×10^{-25}	5.53×10^{31}	2.2×10^{30}
g_7	101.2	1.42×10^{-12}	1.6×10^{33}	1.10×10^{-23}	6.30×10^{31}	2.6×10^{34}

M_{mode} , defined as the ratio of the mode energy E and the average surface velocity of the mode:

$$E = \frac{M_{\text{mode}}}{4\pi} \int \delta v^a \delta v_a^* \sin\theta d\theta d\phi. \quad (3)$$

The mode masses make it possible to convert the observable surface velocities of modes into mode energies.

In our numerical solution of the pulsation equations we use a simple, but somewhat unrealistic, treatment of the surface of the Sun. We set the (Lagrangian) perturbation in the pressure to zero at the point where the pressure vanishes in our solar model. For our equilibrium model we use the realistic equation of state [8] for densities above $10^{-6} \text{ g cm}^{-3}$. For densities below this value we smoothly attach a polytropic ‘‘atmosphere.’’ The mode frequency ω and the gravitational parameter α_{28} are rather insensitive to the choice of density where this artificial atmosphere is attached. However, the mode masses, M_{mode} , which depend on the surface values of the perturbed velocity, are rather more sensitive. We estimate that the errors in our computations of ω and α_{28} are about 0.01%, while the errors in M_{mode} due to our simplified treatment of the surface may be as much as 10%.

III. LISA

LISA is designed to detect the presence of gravitational radiation, or other time-dependent perturbations in the gravitational field, by monitoring the precise distance between pairs of satellites. Consider two satellites which form one arm of the detector. A near-zone gravitational perturbation (such as that produced by the solar oscillations) causes the distance between two satellites to oscillate with the frequency ω of the gravitational perturbation. The amplitude δL of the periodic displacement of the satellites is determined by the standard tidal acceleration formula

$$\omega^2 \delta L = L n^a n^b \nabla_a \nabla_b \delta \Phi. \quad (4)$$

where L is the average separation of the satellites. The unit vector n^a that appears in Eq. (4) gives the direction of the line between the satellites. To facilitate comparison with the published LISA sensitivity curves, we define the dimensionless strain h that would be measured by a detector consisting of two perpendicular interferometer arms:

$$h = \omega^{-2} (n^a n^b - m^a m^b) \nabla_a \nabla_b \delta \Phi, \quad (5)$$

where n^a and m^a are unit vectors pointing in the directions of the two detector arms. The right side of Eq. (5) has a complicated dependence on the orientation of the plane of the detector arms, the orientation of the detector arms within this plane, the position of the center-of-mass of the satellites, and the particular l and m of the mode. For simplicity we will use an average value for this quantity. We first average over all orientations of the two perpendicular arms, at a fixed point in space, to obtain

$$\langle |(n^a n^b - m^a m^b) \nabla_a \nabla_b \delta \Phi|^2 \rangle = \frac{2}{5} \nabla_a \nabla_b \delta \Phi \nabla^a \nabla^b \delta \Phi^*. \quad (6)$$

We now average the right side of Eq. (6) over all possible locations of the satellite around the Sun to find the mean-square value of h :

$$\langle |h|^2 \rangle = \frac{1}{10\pi} \int \nabla_a \nabla_b \delta \Phi \nabla^a \nabla^b \delta \Phi^* \sin\theta d\theta d\phi. \quad (7)$$

Finally using Eq. (1) we evaluate the angular integrals on the right side of Eq. (7) to find the average strain h that would be sensed by the LISA detector:

$$h = \alpha_{28} \sqrt{(l+1)(l+2)(2l+1)(2l+3)} \times \left(\frac{1}{10\pi} \right)^{1/2} \left(\frac{E}{10^{28} \text{ ergs}} \right)^{1/2} \frac{GM_{\odot}}{\omega^2 R_{\odot}^3} \left(\frac{R_{\odot}}{r} \right)^{l+3}. \quad (8)$$

The modes of the Sun with the same n and l but different m have frequencies that are slightly split by the Sun’s rotation. In one year of integration, LISA will be able to distinguish frequencies to within about $\Delta f \approx 3 \times 10^{-2} \mu\text{Hz}$ (for signal-to-noise of order unity), where $f = \omega/2\pi$ is the frequency of the signal. This Δf is roughly an order of magnitude smaller than the size of the splitting due to the Sun’s rotation. Thus modes with different m are effectively nondegenerate, and the energy E that appears in Eq. (8) must be that of a single mode of given l and m .

IV. SIGNAL STRENGTHS AND NOISE LEVELS

To determine whether the oscillation modes of the Sun could be detected by LISA we must determine whether the expected gravitational signals exceed the expected noise in the detector and any ‘‘background’’ gravitational wave signals. We now give estimates for the parameters that determine the signal strengths and the noise levels relevant to this problem.

As we have seen, Eq. (8), the magnitude of the signal in the LISA detector is determined by the energy contained in the modes of the Sun. The modes which are most likely to excite the LISA detector have not been observed to date. This makes the prospects of gravitational observations of these modes very interesting, but it also makes it very difficult to make reliable predictions of the energy contained in these modes. The observed low- l p modes have a maximum energy/mode of roughly 10^{28} ergs for f near $3000\mu\text{Hz}$, with the energy/mode decreasing to roughly 10^{27} ergs at f near $1000\mu\text{Hz}$ [10]. A simple extrapolation of this curve would suggest even lower energies for the modes relevant to LISA ($100\text{--}700\mu\text{Hz}$), but such an extrapolation might well be naive. Goldreich and Murray have argued that the decrease in p mode energy (at fixed l) from 3000 to $1000\mu\text{Hz}$ is due to the scattering of energy from higher-order to lower-order p modes of the same frequency [11]. Clearly, the lowest-order modes p modes will not be damped by this mechanism. Moreover, for fixed mode energy, the lowest-order modes produce significantly larger external gravitational perturbations than the higher-order modes. Thus only the lowest-order p modes are relevant to LISA.

At present only upper limits for the energies of these low-frequency modes are known. This is mostly because, for a given l and mode energy, the detection of modes by Doppler shift measurements becomes much more difficult as one goes to lower frequency modes. This difficulty is due to the fact that the surface velocity (for fixed energy) decreases roughly like $f^{3.5}$ for p modes and f^2 for g modes, while instrumental and background solar velocity noise both increase at lower frequencies [3]. The present observational upper limit on the surface velocities of these modes is about 4 cm s^{-1} [3]. Using the mode masses computed in Table I, this translates into limits on the energies, E_{max} , which are also listed in Table I for each mode. It has been suggested [3] that the low signal-to-noise ‘‘detections’’ of solar g modes are an indication that the g mode energies are in fact just below these maximum values. These energies are rather large, and many solar modes would be well above the LISA noise levels if their energies are close to these values.

Kumar, Quataert, and Bahcall [12] have recently made predictions of the energies contained in the low-frequency solar g modes. They consider the turbulent convection excitation mechanism used by Goldreich, Murray, and Kumar [13] to explain the excitations of the observed p modes. Applying this mechanism to the low- l , low- n g modes, they find that turbulent convection would excite surface velocities of about 10^{-2} cm s^{-1} in these modes, assuming the dissipation time for the modes to be about 10^6 years. These surface velocities correspond (using our computed mode masses) to energies between 2×10^{27} and 2×10^{29} ergs for the g modes listed in Table I. Thus, a typical value for the energy pre-

dicted by this model is about 10^{28} ergs. Kumar and Quataert predict that the low- l , low- n p modes will also have energies of roughly 10^{28} ergs [14].

For frequencies below $1000\mu\text{Hz}$ the anticipated noise in the LISA detector [15] would prevent the detection of periodic signals with amplitudes less than [16]

$$h_{sp} = 3 \times 10^{-22} \left(\frac{f}{100\mu\text{Hz}} \right)^{-2.3}. \quad (9)$$

It is also possible that other gravitational wave sources might produce a background ‘‘noise’’ from which the solar oscillation signals could not easily be distinguished. Hils, Bender, and Webbink [17] have estimated that there are about 3×10^6 short-period white-dwarf binaries in our galaxy, producing a gravitational wave background of about $30 h_{sp}$ in the relevant frequency band. At the time these estimates were made there were no known examples of white-dwarf binaries with periods less than one day. Recently, however, two white-dwarf binaries with periods of 3.47 and approximately 4 h, respectively, were discovered by Marsh [18] and Marsh, Dhillon, and Duck [19]. Given these new observations, it should be possible to make considerably better estimates of the gravitational wave background from binaries, but to our knowledge, such estimates have not yet been completed.

V. RESULTS

Table I lists the values of the strain amplitude h , Eq. (8), for a number of modes of the Sun, assuming that each mode has an energy of 10^{28} ergs and that the detector is located at a distance $r = 1\text{ AU}$ from the Sun. Only the $l = 2$ modes are considered, since the modes with successively higher l values have gravitational signals which are reduced below their $l = 2$ counterparts by additional factors of $R_{\odot}/r \approx 0.005$. And of the $l = 2$ modes, only those of low order are relevant. This might be surprising, since higher-order p modes have higher frequency, while the detector noise decreases like $f^{-2.3}$ by Eq. (9). However, two other factors combine to make the higher-order p modes less favorable for detection. First, for given $\delta\Phi$, the associated strain h scales like f^{-2} , by Eq. (5). Second, the coefficient α_{28} (in effect the quadrupole moment of the mode) decreases as the order of the mode increases, since for higher-order modes the contributions from different radii tend to cancel.

The modes in Table I are listed in order of increasing E_{min} , the mode energy for which $h = 3h_{sp}$. This is the minimum energy needed in a mode so that the mode would be observable above the detector noise with $S/N = 3$. Included in Table I is every $l = 2$ mode with $E_{\text{min}} \leq 10^{32}$ ergs. We see that mode energies of at least 10^{30} ergs are required for any of them to be observable by LISA. This is a factor of (up to) 10^3 smaller than the upper limits, but a factor of 10^2 larger than the energies estimated on theoretical grounds by Kumar, Quataert, and Bahcall [12]. Given these energies, the corresponding surface velocities of these modes would be about 10^{-1} cm s^{-1} , which is roughly the detection threshold for the recently launched SOHO satellite [20]. This indicates that any modes that could be observed by LISA would already have been observed by SOHO. Nevertheless, LISA could still provide some unique information since it mea-

sures the gravitational amplitude of the mode which depends on the density perturbations throughout the interior of the Sun.

If the background radiation due to short-period white dwarf binaries is comparable to the values predicted by Hils, Bender, and Webbink [17], then at $f \approx 300 \mu\text{Hz}$ the gravitational wave background is a factor of about 30 larger than LISA's detector noise. Hence mode energies about 10^3 times greater than the E_{\min} 's listed in Table I would be required for solar oscillations to rise above this background. Such energies would be close to the present upper limits for these modes. Therefore, if the gravitational wave background is close to the strength predicted by Hils, Bender, and Webbink [17], then the mode energies would have to be very close to their maximum allowed values for LISA to contribute at all to helioseismology. LISA's one-year orbit around the Sun

would modulate the solar oscillation signal in a way that differs from the one-year modulation of the background gravitational wave signal. It is conceivable that this difference could be used to help distinguish solar oscillations from the gravitational wave background, but we have not investigated this possibility in any detail.

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