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DAMPING TIMES FOR NEUTRON STAR OSCILLATIONS

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ABSTRACT

The frequencies and damping times due to (bulk and shear) viscosity and gravitational radiation reaction are computed for the lowest frequency $0 \leq l \leq 5$ modes of a wide range of nonrotating fully relativistic neutron star models. These computations generalize previous results by determining the effects of bulk viscosity (using coefficients appropriate for normal matter, pion-condensed matter and strange quark matter, respectively), by examining a large set of modes (including the radial and dipole modes), and by expanding the sample of neutron star matter equations of state. These damping times play a crucial role in determining the stability of rapidly rotating neutron stars (in effect determining their maximum angular velocities) and in any model of periodic astrophysical phenomena that involves a pulsating neutron star.

Subject headings: dense matter — hydrodynamics — stars: neutron — stars: pulsation

I. INTRODUCTION

Pulsars having periods in the millisecond range have been discovered on a regular basis in recent years (see, e.g., Becker *et al.* 1982; Fruchter, Stinebring, and Taylor 1988). These objects are generally believed to be rotating neutron stars whose rotation frequencies are identical to the frequencies of the observed pulsations. It is of fundamental interest, therefore, to determine theoretically the maximum rotation rate that is allowed for a neutron star, and hence the maximum pulsar frequency that can be expected observationally. The maximum angular velocity of a sequence of rotating neutron star models is determined by the point at which the frequency of some mode acquires a negative imaginary part and hence becomes unstable. Thus, an understanding of the frequencies (including their imaginary parts or "damping times") of neutron star oscillations is fundamental. In this paper we explore in some detail one aspect of this problem: the effect of the equation of state of neutron star matter on these oscillations. We present new fully relativistic calculations of the frequencies and damping times (due to bulk and shear viscosity and gravitational radiation) of the $0 \leq l \leq 5$ modes of a wide range of nonrotating neutron star models. In addition to expanding the set of modes previously considered, we explore a larger range of equations of state, including two based on the assumption that there occurs a phase transition to an "exotic" state (pion-condensed matter or quark matter, respectively). We also present new calculations of the bulk viscous damping times.

The stability of young, rapidly rotating neutron stars is probably determined by the balancing of the gravitational radiation reaction forces which threaten to destabilize *all* rotating stars (Chandrasekhar 1970; Friedman and Schutz 1978; Friedman 1978) and viscous forces which insure the stability of sufficiently slowly rotating stars (Lindblom and Detweiler 1977; Lindblom and Hiscock 1983). In order to determine which stars are stable, it is necessary to evaluate the imaginary parts of the frequencies of the relevant modes (the p-modes with $l = m$) to determine whether they are positive or negative. One element of the needed analysis is presented in this paper: the calculation of the frequencies and the damping times (the reciprocal of the imaginary part of the frequency) of nonrotating fully relativistic neutron star models. Another element of this analysis, the angular-velocity dependence of these frequencies and damping times, has at present been carried out only in the context of Newtonian stellar models (Ipser and Lindblom 1989, 1990). While the radial ($l = 0$) and dipole ($l = 1$) modes are not expected to participate in this gravitational instability (since they emit no gravitational radiation in nonrotating stars), these oscillations could perhaps be observed directly, and so we have tabulated their frequencies and damping times as well.

Section II of this paper reviews the methods that we employ to evaluate the frequencies and the gravitational and viscous damping times of the modes of relativistic stellar models. We also correct in that section several errors that appeared in earlier expressions (Cutler and Lindblom 1987) for the shear viscous damping times. Section III reviews the assumptions about the equation of state and the viscous dissipation coefficients that we employ in our computations. Finally, § IV contains the tabulated results of our computations and discusses briefly their significance.

II. EVALUATING THE DAMPING TIMES

This section describes the method that we employ to evaluate the damping times for the modes of nonrotating fully relativistic neutron stars. For neutron stars, the dissipation mechanisms (both viscosity and gravitational radiation) act on the p-modes over time scales that are much longer than the pulsation periods of the modes themselves. It is possible, therefore, to approximate these modes as being essentially adiabatic with small dissipative corrections. We begin with a brief discussion of the method of finding the adiabatic modes of these stars, and then describe in more detail how to include the small effects of viscous dissipation.

The perturbations of a nonrotating relativistic stellar model may be described in terms of the Lagrangian fluid displacement vector, ξ_a, and the perturbed metric tensor, δg_{ab}. The equations of motion for these quantities are derived by linearizing the gravitational field equation about a background equilibrium solution. The solutions of these linearized equations having time dependence $e^{i\omega t}$ are called modes. By the spherical symmetry of the background stellar model, the perturbation quantities may be decomposed into spherical harmonics, Y_l^m, satisfying decoupled equations. Thus, by a suitable choice of gauge, the Lagrangian fluid displacement corresponding to a given mode and spherical harmonic may be written in the form

$$\xi_a = [W(r)e^{\lambda/2}r^{-1}Y_l^m\nabla_a r - V(r)\nabla_a Y_l^m]r^l e^{i\omega t} . \tag{1}$$

Similarly, the perturbed metric tensor may be written in an appropriately chosen gauge as

$$\begin{aligned} ds^2 &= (g_{ab} + \delta g_{ab})dx^a dx^b \\ &= -e^\nu(1 + r^l H_0 Y_l^m e^{i\omega t})dt^2 - 2i\omega r^{l+1}H_1 Y_l^m e^{i\omega t}\,dt\,dr + e^\lambda(1 - r^l H_2 Y_l^m e^{i\omega t})dr^2 + r^2(1 - r^l K Y_l^m e^{i\omega t})(d\theta^2 + \sin^2\theta\,d\phi^2) . \end{aligned} \tag{2}$$

The gauge of the metric can be further specialized by setting $V = H_1 = K = 0$ for the $l = 0$ (radial) oscillations, $K = 0$ for the $l = 1$ (dipole) oscillations, and $H_0 = H_2$ for the $l \geq 2$ oscillations. The equations for H_0, H_1, H_2, K, W, and V (which are functions of r only) that result from imposing the perturbed Einstein equation were first derived for the $l = 0$ modes by Chandrasekhar (1964), for the $l = 1$ modes by Campolattaro and Thorne (1970), and for the $l \geq 2$ modes by Thorne and Campolattaro (1967). These equations, together with appropriate boundary conditions, form an eigenvalue problem for the frequency, ω, of the mode. Because of the emission of gravitational radiation, these frequencies will have nonvanishing imaginary part of the modes with $l \geq 2$ even in the absence of viscosity. Numerical methods for solving this eigenvalue problem for the lowest frequency p-modes (the f-modes when $l \neq 1$) are described in Glass and Lindblom (1983) for $l = 0$, Lindblom and Splinter (1989) for $l = 1$, and in Lindblom and Detweiler (1983) and Detweiler and Lindblom (1985) for $l \geq 2$.

It is relatively straightforward to determine the effect of a small amount of viscous dissipation on the pulsation of a star once ξ_a and δg_{ab} for the dissipation-free mode are known. The presence of dissipation causes the energy E in a mode to decrease according to the formula

$$\frac{dE}{dt} = -\frac{2E}{\tau} . \tag{3}$$

The damping time, τ, for a given mode can be determined, then, once the energy E and the rate of change of the energy dE/dt are known. The rate of change of the energy in a pulsation is related to the dissipation in the fluid by the integral

$$\frac{dE}{dt} = -\int (2\eta\delta\sigma^{ab}\delta\sigma^*_{ab} + \zeta|\delta\sigma|^2)e^\nu e^{\lambda/2}r^2 \sin\theta\,dr\,d\theta\,d\phi - F_G . \tag{4}$$

In this expression $\delta\sigma_{ab}$ and $\delta\sigma$ are the shear and expansion of the perturbed fluid motions, η and ζ are the coefficients of shear and bulk viscosity, and F_G is the flux of energy carried away by gravitational radiation. The rate of change of the energy can be determined, therefore, once the shear and expansion of the mode are known. For small dissipation, $\delta\sigma^{ab}$ and $\delta\sigma$ are essentially the same as those for the corresponding nondissipative mode. Thus, to lowest order in small dissipation coefficients, dE/dt can be determined from equation (4) with only a knowledge of the nondissipative eigenfunctions ξ_a and δg_{ab}. The energy E can similarly be evaluated to lowest order with only a knowledge of the nondissipative mode. For the $l = 0$ and $l = 1$ modes, we use integral expressions for E given in Glass and Lindblom (1983) and Lindblom and Splinter (1989). For the $l \geq 2$ modes, we use the value of the gravitational radiation damping time τ_G (where $1/\tau_G$ is the imaginary part of the frequency of the nonviscous mode) and the gravitational radiation energy flux F_G to evaluate the energy as $E = \frac{1}{2}\tau_G F_G$. These expressions for dE/dt and E are combined as prescribed in equation (3) to determine the viscous damping time τ.

The expression, equation (4), for the time derivative of the energy can be evaluated for any fluid perturbations (ξ_a and δg_{ab}) using formulae given in Lindblom and Hiscock (1983) for the shear and expansion. For perturbations having the forms given in equations (1) and (2), these expressions can be simplified considerably, and the angular integrals can be performed. The resulting damping times (as deduced from eq. [3]) due to the shear and bulk viscosities are given by

$$\frac{1}{\tau_\eta} = \frac{|\omega|^2}{E}\int_0^R \eta r^{2(l-1)}e^{\lambda/2}\left\{\frac{3}{2}(\alpha_1)^2 + 2l(l+1)(\alpha_2)^2 + l(l+1)\left[\frac{1}{2}l(l+1) - 1\right]V^2\right\}dr , \tag{5}$$

and

$$\frac{1}{\tau_\zeta} = \frac{|\omega|^2}{2E}\int_0^R \zeta r^{2(l+1)}e^{\lambda/2}\left\{K + \frac{1}{2}H_2 - \frac{1}{r}e^{-\lambda/2}\left[\frac{dW}{dr} + \frac{1}{r}(l+1)W\right] - l(l+1)\frac{V}{r^2}\right\}^2 dr , \tag{6}$$

where

$$\alpha_1 = \frac{r^2}{3}\left\{\frac{2}{r}e^{-\lambda/2}\left[\frac{dW}{dr} + (l-2)\frac{W}{r}\right] + K - H_2 - l(l+1)\frac{V}{r^2}\right\} , \tag{7}$$

and

$$\alpha_2 = \frac{r}{2}\left[\frac{dV}{dr} + (l-2)\frac{V}{r} - e^{\lambda/2}\frac{W}{r}\right]e^{-\lambda/2} . \tag{8}$$

Given the functions that describe the nonviscous mode of the star (W, V, H_2, and K) it is straightforward, then, to evaluate the viscous damping times using the integrals in equations (5) and (6). An expression of this type for the shear-viscosity damping time first appeared in Cutler and Lindblom (1987). We note that the formulae given there contain several errors which are corrected here. Unfortunately, a missing factor of e^{ν} in their equations (1) and (10) was also omitted from their numerical calculations. The shear-viscosity damping times published in that paper are all too short, therefore, by factors as large as 2 or 3 (although typically far less than this). For this reason, we have reevaluated those shear-viscosity damping times in this paper.

We have neglected in equation (4) a number of other forms of dissipation which may occur in real neutron stars. We have neglected the dissipation due to thermal conductivity, since this effect is roughly 10^6 times smaller than that of shear viscosity for normal neutron star matter (Cutler and Lindblom 1987). Furthermore, the sign of the imaginary part of the frequency for this effect will always be identical to that of the shear viscous term. Magnetic fields could also play a role in damping neutron star oscillations. For magnetic fields that are "frozen in" to the neutron matter, the damping time of the lth mode due to magnetic l-pole radiation is expected to be approximately $\tau_B \approx \omega^{-1}(\rho R^2\omega^2/B^2)(c/R\omega)^{2l+1}$, or $\tau_B \approx 10^9$ s for the $l = 2$ mode with $|\vec{B}| \approx 10^{12}$ G. This time scale is comparable to the viscous time scales but *much* longer than the corresponding gravitational radiation time scale. Furthermore, if the magnetic spin axes are closely aligned, the electromagnetic and gravitational back-reactions both affect the mode in the same way. That is, the imaginary part of the frequency caused by electromagnetic back-reaction will always be the same as the sign of the imaginary part of the frequency due to gravitational back-reaction. (If the magnetic and spin axes are not closely aligned, then the star will spin down on a time scale comparable to the viscous damping time of the perturbation.) Thus, neither thermal conductivity nor magnetic fields are expected to play an interesting role in damping (or amplifying) the oscillations of real neutron stars.

III. MICROPHYSICAL ASSUMPTIONS

In order to evaluate the viscous damping times according to the procedure outlined in § II, the equation of state, $\rho(p)$ (the energy density as a function of the pressure), and the equations for the viscosity coefficients, η and ζ, must be specified. Unfortunately, these microphysical quantities are very poorly known. Numerous theoretical calculations of the equation of state exist in the literature, but they do not agree well with each other. In order to estimate the effect of this uncertainty on the dynamical properties of neutron stars, we evaluate the modes of neutron stars based on a representative sample of the published equations of state. We also use several different expressions for the dissipation coefficients, corresponding to different temperature regimes and to different possible phases of neutron star matter.

We construct neutron star models using a sample of 12 equations of state. These equations were chosen to illustrate the range of published results; however, we have excluded equations of state from our sample that are so soft that they fail to have stellar models with masses larger than 1.4 $M_\odot$. We use a number of equations of state that were included in the study of the properties of neutron stars by Arnett and Bowers (1974, 1977): $P_N(A)$, the Pandharipande (1971) pure neutron equation that is based on the Reid soft-core nuclear interaction; $P_H(B)$, the Pandharipande (1971) equation that includes hyperonic matter; $BJ_I(C)$, the Bethe and Johnson (1974) (model I) equation based on an improved neutron-neutron potential; A(F), the Arponen (1972) equation based on a Thomas-Fermi model of the interactions; and MF(L), the Pandharipande and Smith (1975) equation based on a mean-field effective scalar-meson interaction. (The letter in parentheses following each reference refers to the Arnett and Bowers terminology for that model.) We also include five more recent equations of state that describe "normal" nuclear matter: RMF, the Serot (1979*a*) equation based on a self-consistent relativistic mean-field effective scalar-meson interaction; WFF, the Wiringa, Fiks, and Fabrocini (1988) equation (the UV14 plus TNI model) that includes three-nucleon interactions; G_{210}, G_{240}, and G_{300}, the Glendenning (1986) parameterized field theoretical models for three values of the compressibility modulus, K = 210, 240, and 300 MeV. Finally, we include two equations of state describing nuclear matter than has undergone a phase transition to an "exotic" state: π, the Maxwell and Weise (1976) equation (with $g_A^* = 1.3$) which includes pion condensation; and q, the Glendenning (1989) equation describing strange quark matter (with bag constant $B^{1/4}$ = 170 MeV). These equations of state are summarized in Table 1.

In order to evaluate the damping times with equations (5) and (6), expressions for the viscosity coefficients η and ζ must also be specified. For the case of shear viscosity, η, two different physical regimes are anticipated in the neutron star interior. At high enough

TABLE 1

EQUATIONS OF STATE

Equation of State	Reference	Description
q	Glendenning 1989	Quark matter
RMF	Serot 1979	Relativistic mean field
MF(L)	Pandharipande and Smith 1975	Mean field
WFF	Wiringa, Fiks, and Fabrocini 1988	Three-nucleon interactions
π	Maxwell and Weise 1976	Pion condensation
$P_N(A)$	Pandharipande 1971	Reid soft core, pure neutrons
$P_H(B)$	Pandharipande 1971	Reid soft core, with hyperons
G_{210}, G_{240}, G_{300}	Glendenning 1986	Parameterized compressability
$BJ_I(C)$	Bethe and Johnson 1974	Pure neutrons, model I
A(F)	Arponen 1972	Thomas-Fermi interaction

temperatures, the viscosity will be produced primarily by the scattering of neutrons. At sufficiently low temperature ($T \lesssim 10^9$ K) the neutrons and protons become superfluid, and electron scattering becomes the dominant dissipative mechanism. The shear viscosity η_n for the high-temperature regime has been calculated by Flowers and Itoh (1976). We use the following analytical fit to their numerical results:

$$\eta_n = 1.95 \times 10^{18} \frac{\rho_{15}{}^{9/4}}{T_9{}^2}, \tag{9}$$

where η_n has units g cm^{-1} s^{-1}, ρ_{15} has units 10^{15} g cm^{-3}, and T_9 has units 10^9 K. This formula, equation (9), reproduces the Flowers and Itoh (1976) computations to within about 3%. The viscosity, η_e, due to electron scattering (which is appropriate for low-temperature neutron stars) is given approximately by

$$\eta_e = 6.0 \times 10^{18}\left(\frac{\rho_{15}}{T_9}\right)^2. \tag{10}$$

This formula reproduces the exact electron scattering result to within about 5%. We note that η_e is larger than η_n for typical neutron star densities because the electron mean free path is substantially increased when the neutrons and protons become superfluid. In this paper we calculate a "normal" shear viscous time scale, setting $\eta = \eta_n$ everywhere, and a "superfluid" shear viscous time scale, setting $\eta = \eta_e$ everywhere. A somewhat more realistic treatment would take the neutron star as having a superfluid region and a normal region, with η_e used as the dissipation coefficient in the former and η_n in the latter (see Cutler and Lindblom 1987). This refinement would not substantially change the value of the computed damping times, however, except for a narrow range of temperatures near the superfluid critical temperature.

Neutron star matter also dissipates energy via bulk viscosity. For neutron stars, the dynamical oscillation time scale is much shorter than the time scale required for the nuclear matter to return to complete equilibrium. The variations of the pressure and density in the perturbed fluid become out of phase therefore. This phase lag results in energy being dissipated from the pulsations via $P\,dV$ work. The nuclear matter therefore acquires a (frequency-dependent) bulk viscosity that scales roughly as the square of the pulsation period over the equilibration time scale. Sawyer (1989*a*) derives the following expression for ζ_n, the bulk viscosity for normal neutron star matter (in the high-frequency limit):

$$\zeta_n = 6.0 \times 10^{25}\left(\frac{\rho_{15}}{\omega}\right)^2 e^{\nu} T_9{}^6, \tag{11}$$

where ζ_n has the units g cm^{-1} s^{-1} and ω is the frequency of the mode in s^{-1}. The factor e^{ν} which appears in equation (11) transforms ω into the proper frequency of the mode as observed locally by an element of the stellar fluid. When the neutron star matter has undergone a phase transition into an "exotic" state, the particle interactions available to the material are changed. The equilibration time scales for both pion-condensed matter and quark matter are much shorter than for normal neutron star matter, and their bulk viscosity coefficients are correspondingly larger. The following bulk viscosity coefficient for pion-condensed matter, ζ_π, has been derived by Sawyer (1989*b*):

$$\zeta_\pi = 5.6 \times 10^{29} \frac{\sin(2\theta) n^{8/3} T_9{}^4}{e^{-\nu}\omega^2}, \tag{12}$$

where n is the baryon number density given in units of fm^{-3}. The overall coefficient in equation (12) is proportional to a parameter $\sin(2\theta)$ that measures the degree of pion condensation. (The definition of θ is essentially that of Baym *et al.* 1975.) For simplicity, we set $\sin(2\theta) = \frac{1}{2}$ for densities above $\rho = 5.0 \times 10^{14}$ g cm^{-3} (the density above which pion condensation occurs), and we take the bulk viscosity coefficient to be ζ_n below that density. The bulk viscosity coefficient of strange quark matter, ζ_q, has also been computed by Sawyer (1989*c*) and is well-approximated (to within a few percent) as

$$\zeta_q = 7.07 \times 10^{33} \frac{(n - 0.03) T_9{}^2}{e^{-\nu}\omega^2 + 1.7 \times 10^6 n^2 T_9{}^4}. \tag{13}$$

This expression assumes a strange quark mass of $M_S = 100$ MeV.

IV. RESULTS AND DISCUSSION

The pulsations and damping times have been computed for the modes of neutron stars constructed from each of the 12 equations of state listed in Table 1. For each equation of state, we examine the properties of three different stellar models: the model containing 1.4 $N_\odot$ baryons (where $N_\odot = 1.19 \times 10^{57}$ is the number of baryons in the Sun), the model with mass equal to 1.4 $M_\odot$ (where "mass" refers to the gravitational mass), and the model having the largest possible mass for the equation of state. The minimum number of baryons that are capable of undergoing gravitational collapse to form a neutron star is approximately 1.4 $N_\odot$; thus, this model has approximately the minimum possible mass. Table 2 lists the basic parameters characterizing the equilibrium stellar models constructed from each of the equations of state: the mass M, the radius R, the number of baryons N, the central density ρ_c, the central gravitational potential e^{ν_c}, the surface redshift $z = (1 - 2GM/c^2R)^{-1/2} - 1$, and the fundamental frequency, Ω_0 defined by

$$\Omega_0 = \left(\frac{3GM}{4R^3}\right)^{1/2}. \tag{14}$$

TABLE 2

EQUILIBRIUM STELLAR MODELS

Equation of State	$M/M_{\odot}$	R (km)	$N/N_{\odot}$	Central Density (ρ_{15})	e^{ν_c}	Surface Redshift (z)	Ω_0 (10^4 s^{-1})
q	1.230	8.438	1.400	1.516	0.366	0.3249	1.428
	1.400	8.518	1.627	2.005	0.283	0.3938	1.502
	1.499	8.173	1.765	3.539	0.186	0.4768	1.653
RMF	1.291	13.535	1.400	0.5037	0.524	0.1798	0.7199
	1.400	13.621	1.529	0.5346	0.492	0.1982	0.7426
	2.571	12.507	3.096	1.737	0.0952	0.5949	1.144
MF(L)	1.304	14.939	1.400	0.4175	0.549	0.1607	0.6240
	1.400	14.985	1.512	0.4397	0.523	0.1751	0.6436
	2.661	13.638	3.159	1.484	0.118	0.5358	1.022
WFF	1.269	10.876	1.400	1.077	0.418	0.2351	0.9909
	1.400	10.813	1.565	1.212	0.365	0.2723	1.050
	1.840	9.487	2.164	3.167	0.111	0.5296	1.465
π	1.243	8.325	1.400	2.666	0.260	0.3373	1.464
	1.400	7.903	1.618	3.447	0.165	0.4479	1.680
	1.483	7.278	1.743	5.535	0.0800	0.5842	1.957
$\mathrm{P}_N(A)$	1.267	10.042	1.400	1.659	0.350	0.2624	1.116
	1.400	9.733	1.571	1.977	0.280	0.3184	1.229
	1.626	8.260	1.885	4.386	0.0883	0.5452	1.695
G_{300}	1.296	13.521	1.400	0.6881	0.497	0.1810	0.7224
	1.400	13.360	1.523	0.8109	0.453	0.2033	0.7645
	1.708	11.079	1.909	2.490	0.203	0.3547	1.118
$\mathrm{P}_H(B)$	1.232	8.183	1.400	2.918	0.253	0.3417	1.496
	1.400	7.359	1.639	4.875	0.115	0.5104	1.870
	1.413	7.040	1.658	6.026	0.0815	0.5666	2.008
$\mathrm{BJ}_I(C)$	1.300	12.848	1.400	0.9740	0.442	0.1941	0.7811
	1.400	12.534	1.521	1.102	0.395	0.2215	0.8413
	1.850	9.967	2.111	3.142	0.108	0.4874	1.364
G_{240}	1.297	13.179	1.400	0.8644	0.473	0.1872	0.7510
	1.400	12.768	1.524	1.101	0.414	0.2160	0.8183
	1.594	10.553	1.769	2.846	0.204	0.3434	1.162
A(F)	1.262	10.145	1.400	1.933	0.347	0.2571	1.097
	1.400	8.980	1.581	3.220	0.205	0.3612	1.387
	1.456	7.825	1.660	5.239	0.0978	0.4896	1.739
G_{210}	1.296	12.543	1.400	1.198	0.432	0.1996	0.8086
	1.400	11.559	1.527	1.785	0.336	0.2476	0.9499
	1.456	10.180	1.599	3.143	0.222	0.3156	1.172

The equations of state in these tables have been ordered by the ratio of the central density to the average density in the 1.4 $M_{\odot}$ stellar model. Large values of this ratio arise in "soft" equations of state [such as A(F) or G_{210}], while smaller values of this ratio occur in "stiff" equations of state (such as q or RMF).

Tables 3 and 4 list the frequencies and gravitational radiation damping times for the modes of these stars. Each of these frequencies is given in terms of the fundamental frequency Ω_0. The frequencies and damping times were computed numerically using the techniques outlined in § II. The eigenvalue problem was solved on a numerical grid containing at least 2000 points inside the stellar model. We estimate the accuracy of the frequencies to be about 0.1% and the accuracy of the damping times to be about 1%. The modes considered here are the lowest frequency p-modes for each value of l: $0 \leq l \leq 5$. For $l \neq 1$ these are the f-modes having no nodes in $W(r)$, the radial Lagrangian displacement. For $l = 1$ the lowest frequency mode has one node in $W(r)$ and corresponds to the p_1-mode. Because of spherical symmetry, the frequencies and damping times of these modes are independent of the spherical harmonic index m. The frequency of the radial ($l = 0$) mode is not listed in Table 3 for the maximum mass model of each equation of state, since the frequency vanishes for these marginally unstable models. Damping times are not listed in Table 4 for the radial ($l = 0$) and the dipole ($l = 1$) modes since these modes do not couple to gravitational radiation.

The damping times due to shear viscosity are presented in Tables 5 and 6. They are evaluated using the expressions presented in § III. The normal fluid damping times in Table 5 were computed with the assumption that the shear viscosity $\eta = \eta_n$ from equation (9), while in Table 6 the superfluid damping times were computed under the assumption that $\eta = \eta_e$ from equation (10). In all of our calculations we assume that the neutron star is in thermal equilibrium; i.e., the "redshifted temperature" $Te^{\nu/2}$ is uniform throughout the star. The results given in Tables 5 and 6 are for the central temperature $T_9 = 1.0$. These damping times scale with temperature as T^2. We have not included calculations for the quark matter equation of state, q, since the above expressions for the shear viscosity are not appropriate for neutron star matter in that state.

TABLE 3

Oscillation Frequencies

Equation of State	$M/M_{\odot}$	ω/Ω_0 $l = 0$	$l = 1$	$l = 2$	$l = 3$	$l = 4$	$l = 5$
q	1.230	1.691	3.416	1.073	1.457	1.762	2.023
	1.400	1.128	2.753	1.079	1.447	1.740	1.992
	1.499	...	2.039	1.096	1.442	1.721	1.962
RMF	1.291	2.777	4.222	1.283	1.763	2.136	2.450
	1.400	2.583	4.071	1.268	1.736	2.100	2.409
	2.571	...	2.046	1.113	1.456	1.734	1.973
MF(L)	1.304	2.763	4.088	1.336	1.834	2.217	2.540
	1.400	2.625	3.991	1.319	1.806	2.183	2.500
	2.661	...	2.117	1.149	1.503	1.788	2.033
WFF	1.269	2.078	3.405	1.288	1.740	2.090	2.387
	1.400	1.866	3.212	1.266	1.700	2.038	2.325
	1.840	...	2.040	1.168	1.522	1.806	2.051
π	1.243	1.632	2.870	1.303	1.725	2.053	2.328
	1.400	1.210	2.561	1.229	1.615	1.921	2.181
	1.483	...	2.081	1.157	1.512	1.797	2.040
$P_N(A)$	1.267	1.685	2.947	1.343	1.786	2.127	2.414
	1.400	1.470	2.749	1.305	1.724	2.050	2.326
	1.626	...	2.063	1.190	1.545	1.830	2.073
G_{300}	1.296	1.920	3.409	1.351	1.831	2.200	2.511
	1.400	1.623	3.172	1.339	1.802	2.160	2.463
	1.708	...	2.079	1.321	1.703	2.006	2.266
$P_H(B)$	1.232	1.404	2.686	1.296	1.708	2.029	2.301
	1.400	0.6458	2.182	1.202	1.567	1.858	2.106
	1.413	...	2.003	1.176	1.530	1.813	2.055
$BJ_I(C)$	1.300	1.733	2.912	1.440	1.909	2.263	2.559
	1.400	1.635	2.821	1.415	1.869	2.214	2.503
	1.850	...	2.046	1.245	1.610	1.900	2.147
G_{240}	1.297	1.542	3.113	1.376	1.848	2.211	2.517
	1.400	1.240	2.804	1.369	1.820	2.168	2.463
	1.594	...	2.078	1.351	1.734	2.037	2.297
A(F)	1.262	1.182	2.676	1.354	1.788	2.124	2.410
	1.400	0.7972	2.198	1.319	1.707	2.013	2.275
	1.456	...	1.907	1.254	1.611	1.896	2.139
G_{210}	1.296	1.140	2.744	1.405	1.862	2.213	2.511
	1.400	0.8777	2.396	1.403	1.826	2.155	2.435
	1.456	...	2.087	1.395	1.783	2.088	2.350

TABLE 4

Gravitational Damping Times

Equation of State	$M/M_{\odot}$	$\frac{\tau_G \Omega_0}{10^3}$ $l = 2$	$\frac{\tau_G \Omega_0}{10^5}$ $l = 3$	$\frac{\tau_G \Omega_0}{10^7}$ $l = 4$	$\frac{\tau_G \Omega_0}{10^9}$ $l = 5$
q	1.230	1.86	1.51	1.47	1.18
	1.400	1.73	1.39	1.34	1.05
	1.499	1.73	1.43	1.42	1.18
RMF	1.291	2.50	2.25	2.42	1.94
	1.400	2.28	1.99	2.08	1.69
	2.571	2.10	1.90	2.02	1.92
MF(L)	1.304	2.70	2.51	2.81	2.39
	1.400	2.47	2.23	2.43	2.05
	2.661	1.88	1.66	1.73	1.57
WFF	1.269	1.87	1.57	1.60	1.28
	1.400	1.71	1.40	1.40	1.10
	1.840	1.87	1.66	1.75	1.59
π	1.243	1.50	1.23	1.25	1.03
	1.400	1.63	1.39	1.43	1.30
	1.483	2.16	2.01	2.20	2.06
$P_N(A)$	1.267	1.62	1.36	1.39	1.13
	1.400	1.53	1.26	1.28	1.03
	1.626	2.00	1.84	1.98	1.90
G_{300}	1.296	2.27	2.05	2.24	1.90
	1.400	2.03	1.78	1.89	1.57
	1.708	1.51	1.32	1.36	1.14
$P_H(B)$	1.232	1.51	1.25	1.27	1.04
	1.400	1.82	1.63	1.73	1.60
	1.413	2.08	1.93	2.11	2.04
$BJ_I(C)$	1.300	1.91	1.76	1.98	1.69
	1.400	1.73	1.54	1.68	1.44
	1.850	1.78	1.62	1.74	1.71
G_{240}	1.297	2.13	1.94	2.12	1.79
	1.400	1.85	1.64	1.75	1.48
	1.594	1.49	1.33	1.40	1.18
A(F)	1.262	1.63	1.40	1.45	1.17
	1.400	1.50	1.31	1.35	1.13
	1.456	1.83	1.70	1.84	1.72
G_{210}	1.296	1.93	1.77	1.94	1.63
	1.400	1.61	1.45	1.55	1.25
	1.456	1.48	1.35	1.45	1.20

TABLE 5

SHEAR VISCOUS DAMPING TIMES: NORMAL FLUID

Equation of State	$M/M_\odot$	$\frac{\tau_\eta \Omega_0}{10^{14}}$ $l=0$	$\frac{\tau_\eta \Omega_0}{10^{11}}$ $l=1$	$\frac{\tau_\eta \Omega_0}{10^{11}}$ $l=2$	$\frac{\tau_\eta \Omega_0}{10^{11}}$ $l=3$	$\frac{\tau_\eta \Omega_0}{10^{11}}$ $l=4$	$\frac{\tau_\eta \Omega_0}{10^{11}}$ $l=5$
RMF	1.291	1.76	43.8	17.5	7.92	4.70	3.26
	1.400	1.11	47.2	17.5	7.95	4.72	3.21
	2.571	...	52.0	6.71	3.29	2.01	1.38
MF(L)	1.304	2.61	51.9	23.3	10.9	6.66	4.69
	1.400	2.49	55.0	23.2	10.9	6.65	4.64
	2.661	...	57.8	9.57	4.83	3.00	2.09
WFF	1.269	1.10	21.9	6.72	3.30	2.05	1.44
	1.400	0.780	22.8	6.28	3.11	1.94	1.37
	1.840	...	17.1	2.53	1.33	0.843	0.598
π	1.243	0.111	5.93	1.83	0.956	0.640	0.489
	1.400	0.134	7.32	1.38	0.712	0.465	0.338
	1.483	...	6.98	0.806	0.409	0.262	0.195
$P_N(A)$	1.267	4.95	14.6	4.00	2.17	1.46	1.10
	1.400	2.48	14.4	3.39	1.84	1.24	0.930
	1.626	...	11.4	1.30	0.707	0.469	0.346
G_{300}	1.296	0.626	36.9	14.6	7.45	4.75	3.39
	1.400	0.391	35.6	13.4	6.99	4.49	3.21
	1.708	...	17.3	4.54	2.92	2.06	1.54
$P_H(B)$	1.232	0.156	7.07	1.77	0.962	0.649	0.487
	1.400	0.0720	6.79	0.963	0.517	0.343	0.232
	1.413	...	6.22	0.720	0.385	0.254	0.188
$BJ_I(C)$	1.300	6.31	29.9	9.19	5.48	3.99	3.21
	1.400	8.40	28.2	8.18	4.86	3.53	2.79
	1.850	...	16.6	2.51	1.45	1.02	0.764
G_{240}	1.297	0.350	31.1	12.2	6.64	4.38	3.21
	1.400	0.343	27.1	10.1	5.76	3.87	2.82
	1.594	...	13.8	3.61	2.42	1.78	1.38
$A(F)$	1.262	0.245	13.4	4.10	2.37	1.61	1.20
	1.400	0.276	10.4	2.16	1.34	0.950	0.726
	1.456	...	7.34	1.03	0.636	0.457	0.352
G_{210}	1.296	0.293	22.4	9.07	5.44	3.76	2.82
	1.400	0.401	17.8	5.82	3.83	2.78	2.16
	1.456	...	10.8	3.05	2.17	1.66	1.33

TABLE 6

SHEAR VISCOUS DAMPING TIMES: SUPERFLUID

Equation of State	$M/M_\odot$	$\frac{\tau_\eta \Omega_0}{10^{14}}$ $l=0$	$\frac{\tau_\eta \Omega_0}{10^{11}}$ $l=1$	$\frac{\tau_\eta \Omega_0}{10^{11}}$ $l=2$	$\frac{\tau_\eta \Omega_0}{10^{11}}$ $l=3$	$\frac{\tau_\eta \Omega_0}{10^{11}}$ $l=4$	$\frac{\tau_\eta \Omega_0}{10^{11}}$ $l=5$
RMF	1.291	0.383	10.5	4.33	1.89	1.09	0.741
	1.400	0.267	11.6	4.36	1.91	1.11	0.737
	2.571	...	17.9	1.91	0.881	0.519	0.384
MF(L)	1.304	0.459	11.3	5.46	2.45	1.45	1.00
	1.400	0.494	12.3	5.49	2.47	1.46	0.998
	2.661	...	19.1	2.61	1.24	0.739	0.501
WFF	1.269	0.264	6.39	1.94	0.906	0.545	0.372
	1.400	0.208	6.89	1.85	0.869	0.524	0.359
	1.840	...	6.80	0.829	0.404	0.246	0.169
π	1.243	0.0340	1.93	0.646	0.319	0.204	0.150
	1.400	0.0458	2.65	0.507	0.246	0.154	0.108
	1.483	...	3.06	0.306	0.146	0.0896	0.0646
$P_N(A)$	1.267	1.40	4.62	1.25	0.635	0.408	0.296
	1.400	0.774	4.80	1.09	0.555	0.355	0.257
	1.626	...	4.65	0.460	0.231	0.146	0.103
G_{300}	1.296	0.159	9.47	3.70	1.78	1.09	0.757
	1.400	0.101	9.81	3.46	1.70	1.05	0.727
	1.708	...	6.42	1.39	0.797	0.526	0.377
$P_H(B)$	1.232	0.0492	2.52	0.627	0.318	0.204	0.147
	1.400	0.0270	2.83	0.364	0.182	0.115	0.0747
	1.413	...	2.77	0.277	0.137	0.0862	0.0613
$BJ_I(C)$	1.300	1.53	8.18	2.50	1.38	0.942	0.722
	1.400	2.06	7.96	2.28	1.25	0.855	0.645
	1.850	...	6.31	0.818	0.433	0.286	0.205
G_{240}	1.297	0.0902	8.78	3.18	1.61	1.02	0.721
	1.400	0.0882	8.21	2.72	1.44	0.920	0.646
	1.594	...	5.23	1.14	0.679	0.463	0.341
$A(F)$	1.262	0.0718	4.65	1.27	0.677	0.437	0.315
	1.400	0.0848	4.06	0.735	0.413	0.275	0.201
	1.456	...	3.14	0.377	0.210	0.141	0.103
G_{210}	1.296	0.0755	6.93	2.48	1.36	0.888	0.642
	1.400	0.104	6.01	1.72	1.01	0.686	0.511
	1.456	...	4.18	0.983	0.618	0.435	0.331

The damping times due to bulk viscosity are presented in Table 7. We take the bulk viscosity coefficient $\zeta = \zeta_n$ from equation (11) for all of the neutron star models except those constructed from the quark matter and pion-condensed matter in equations of state q and π. All of these "normal" bulk viscous damping times were evaluated for stellar models having central temperature $T_9 = 1.0$. In these models the damping times scale with temperature as T^{-6}. For the stellar models constructed from equation of state q we take $\zeta = \zeta_q$ from equation (13). The damping times for these models do not scale simply with temperature, so we give the damping times for two central temperatures: $T_9 = 1.0$ and 10.0. For the stellar models constructed from equation of state π, we take $\zeta = \zeta_\pi$ from equation (12) in the inner core of the star, where $\rho \geq 5 \times 10^{14}$ g cm^{-3} and $\zeta = \zeta_n$ from equation (11) below that density. While the damping times for the models containing pion condensation do not scale simply with temperature, we find that they do in fact scale as T^{-4} to within a few percent for temperatures below $T_9 = 10.0$. Consequently, for equation of state π, we tabulate the damping times only for $T_9 = 1.0$

Consider first the implications of these computations on the modes ($l \geq 2$) that determine the stability of rapidly rotating neutron stars. The ratio of the bulk to the shear viscosity damping times, τ_ζ/τ_η, for normal nuclear matter scales with temperature is T^{-8}. Our numerical computations show that this ratio has values in the range 10^4–10^7 at $T_9 = 1.0$. Thus the bulk viscosity will be insignificant compared to the shear viscosity for temperatures below 3–8 $\times 10^9$ K. The cooling calculations of Nomoto and Tsuruta (1987) indicate that the central temperature of a neutron star will fall below these values within about 10^5 s of its birth. Thus, bulk viscosity will only play a significant role in affecting the stability of these modes for about the first day of a neutron star's existence.

TABLE 7

BULK VISCOUS DAMPING TIMES

Equation of State	$M/M_\odot$	$\tau_\zeta \Omega_0$: $l=0$	$l=1$	$l=2$	$l=3$	$l=4$	$l=5$
q ($T_9 = 1$)	1.230	1.01×10^6	5.78×10^6	1.86×10^{10}	2.56×10^{10}	3.81×10^{10}	5.52×10^{10}
	1.400	8.20×10^5	7.91×10^6	8.30×10^9	1.24×10^{10}	1.95×10^{10}	2.99×10^{10}
	1.499	...	1.39×10^7	2.72×10^9	4.92×10^9	8.60×10^9	1.41×10^{10}
q ($T_9 = 10$)	1.230	4.97×10^4	1.08×10^5	1.35×10^9	1.04×10^9	1.12×10^9	1.32×10^9
	1.400	7.07×10^4	1.68×10^5	4.64×10^8	4.01×10^8	4.66×10^8	5.90×10^8
	1.499	...	3.49×10^5	9.85×10^7	1.10×10^8	1.49×10^8	2.11×10^8
RMF	1.291	5.93×10^{13}	1.86×10^{14}	8.07×10^{16}	5.23×10^{16}	4.20×10^{16}	3.71×10^{16}
	1.400	6.43×10^{13}	2.30×10^{14}	1.00×10^{17}	6.70×10^{16}	5.48×10^{16}	4.83×10^{16}
	2.571	...	7.55×10^{15}	1.94×10^{19}	1.20×10^{19}	9.99×10^{18}	9.65×10^{18}
MF(L)	1.304	4.79×10^{13}	1.40×10^{14}	3.55×10^{16}	2.25×10^{16}	1.79×10^{16}	1.55×10^{16}
	1.400	5.26×10^{13}	1.69×10^{14}	4.36×10^{16}	2.82×10^{16}	2.27×10^{16}	1.98×10^{16}
	2.661	...	4.55×10^{15}	4.26×10^{18}	3.14×10^{18}	2.94×10^{18}	2.96×10^{18}
WFF	1.269	4.83×10^{13}	2.30×10^{14}	4.69×10^{16}	3.60×10^{16}	3.35×10^{16}	3.28×10^{16}
	1.400	5.73×10^{13}	3.31×10^{14}	6.98×10^{16}	5.49×10^{16}	5.19×10^{16}	5.20×10^{16}
	1.840	...	3.94×10^{15}	1.82×10^{18}	1.53×10^{18}	1.59×10^{18}	1.72×10^{18}
π	1.243	3.99×10^{10}	2.87×10^{11}	4.82×10^{13}	3.19×10^{13}	2.77×10^{13}	2.74×10^{13}
	1.400	5.84×10^{10}	7.16×10^{11}	2.34×10^{14}	1.37×10^{14}	1.11×10^{14}	1.01×10^{14}
	1.483	...	2.54×10^{12}	2.17×10^{15}	1.25×10^{15}	8.79×10^{14}	7.67×10^{14}
$P_N(A)$	1.267	3.84×10^{13}	2.60×10^{14}	2.16×10^{16}	1.70×10^{16}	1.61×10^{16}	1.63×10^{16}
	1.400	5.31×10^{13}	4.67×10^{14}	4.45×10^{16}	3.56×10^{16}	3.42×10^{16}	3.50×10^{16}
	1.626	...	4.53×10^{15}	2.36×10^{18}	1.52×10^{18}	1.40×10^{18}	1.42×10^{18}
G_{300}	1.296	2.48×10^{13}	1.45×10^{14}	1.85×10^{16}	1.56×10^{16}	1.50×10^{16}	1.48×10^{16}
	1.400	2.16×10^{13}	1.76×10^{14}	1.96×10^{16}	1.85×10^{16}	1.90×10^{16}	1.95×10^{16}
	1.708	...	8.10×10^{14}	2.82×10^{16}	4.71×10^{16}	7.59×10^{16}	1.10×10^{17}
$P_H(B)$	1.232	5.25×10^{13}	4.90×10^{14}	5.23×10^{16}	4.06×10^{16}	3.95×10^{16}	4.14×10^{16}
	1.400	6.34×10^{13}	3.30×10^{15}	1.02×10^{18}	6.34×10^{17}	5.61×10^{17}	5.12×10^{17}
	1.413	...	6.82×10^{15}	3.69×10^{18}	2.13×10^{18}	1.76×10^{18}	1.71×10^{18}
$BJ_I(C)$	1.300	2.27×10^{13}	1.43×10^{14}	5.77×10^{15}	4.96×10^{15}	5.24×10^{15}	5.96×10^{15}
	1.400	2.78×10^{13}	1.95×10^{14}	8.25×10^{15}	7.07×10^{15}	7.43×10^{15}	8.30×10^{15}
	1.850	...	4.33×10^{15}	5.77×10^{17}	4.48×10^{17}	4.46×10^{17}	4.67×10^{17}
G_{240}	1.297	1.53×10^{13}	1.38×10^{14}	1.11×10^{16}	1.09×10^{16}	1.14×10^{16}	1.21×10^{16}
	1.400	1.32×10^{13}	1.71×10^{14}	1.05×10^{16}	1.24×10^{16}	1.47×10^{16}	1.65×10^{16}
	1.594	...	7.51×10^{14}	2.03×10^{16}	3.23×10^{16}	5.14×10^{16}	7.60×10^{16}
A(F)	1.262	1.61×10^{13}	2.26×10^{14}	1.34×10^{16}	1.51×10^{16}	1.76×10^{16}	2.01×10^{16}
	1.400	2.53×10^{13}	7.39×10^{14}	3.26×10^{16}	4.19×10^{16}	5.72×10^{16}	7.50×10^{16}
	1.456	...	4.61×10^{15}	3.59×10^{17}	3.49×10^{17}	4.41×10^{17}	5.78×10^{17}
G_{210}	1.296	8.53×10^{12}	1.31×10^{14}	6.08×10^{15}	7.85×10^{15}	9.81×10^{15}	1.16×10^{16}
	1.400	8.98×10^{12}	2.20×10^{14}	6.17×10^{15}	9.35×10^{15}	1.40×10^{16}	1.93×10^{16}
	1.456	...	5.47×10^{14}	1.07×10^{16}	1.75×10^{16}	2.90×10^{16}	4.51×10^{16}

The second area in which these computations are likely to have interesting implications is the damping of the radial ($l = 0$) modes. For radial oscillations, we find that the bulk viscous damping time scale for normal matter is comparable to the shear viscous damping times at $T_9 = 1$: $\tau_\zeta \approx \tau_\eta$. Each of the bulk viscous damping times for normal nuclear matter lies in the range 1–9 $\times$ 10^9 s at this temperature. The damping times will be shorter than this at both higher temperatures (due to bulk viscosity) and at lower temperatures (due to shear viscosity). The bulk viscosity damping times scale with temperature as T^{-6}. Thus, at $T_9 = 10$ the damping time is reduced to $\sim 10^3$ s, and at $T_9 = 100$ it is reduced further to $\sim 10^{-3}$ s. According to the calculations of Burrows and Lattimer (1986), it takes ~ 5 s for the newly formed neutron star's central temperature to cool from $T_9 = 200$ to $T_9 = 100$. Thus, the radial pulsations that were excited during the initial formation of the neutron star would be completely damped out.

We remark that the bulk viscous damping times calculated here for the $2 \leq l \leq 5$ modes are far longer—by factors ranging between 10 and 10^4 at $T_9 = 1$—than would have been estimated using the back-of-the-envelope calculation given in Cutler and Lindblom (1987). The most important source of error in that estimate was the assumption that the bulk viscosity coefficient was uniform throughout the star. That approximation overestimates the effect of the bulk viscous damping for basically two reasons. First, the bulk viscosity coefficient, ζ_n, is proportional to the density squared, equation (11). The integrand in equation (6) is largest when the density is only 1/10 to 1/3 of its central value. This effect increases the damping times by a factor of 10–100. Second, the bulk viscosity coefficient is proportional to the temperature to the sixth power. For neutron stars in thermal equilibrium (as assumed here) the temperature is smaller in the outer regions of the star due to redshift effects. We find that the temperature at the maximum of the integrand of equation (6) has values between 1/2 and 9/10 of the central value. This effect increases the damping time by an additional factor of 2–100.

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