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THE ACCURACY OF THE RELATIVISTIC COWLING APPROXIMATION

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ABSTRACT

We investigate the accuracy of two versions of the relativistic Cowling approximation for the oscillations of general relativistic stellar models: one by McDermott, Van Horn and Scholl, and a refinement by Finn. The oscillation frequencies and the viscous damping time scales of several of the lowest frequency dipole p-modes in these approximations are compared to the exact values for these quantities. We find, as expected, that the accuracy of the Cowling approximation improves as the number of nodes in the eigenfunctions of the mode increases. Furthermore, the McDermott, Van Horn, and Scholl version of the Cowling approximation is more accurate than Finn's refined version for the low-order p-modes studied here.

Subject headings: relativity — stars: pulsation

I. INTRODUCTION

Cowling's (1941) study of the nonradial pulsations of Newtonian stars found that the perturbations in the gravitational potential could be neglected in the linearized pulsation equations when considering high-order p- and g-modes. McDermott, Van Horn, and Scholl (1983) extended the Cowling approximation to relativistic stellar pulsations by neglecting the perturbations in the metric, δg_{ab}, (which describes the gravitational field in general relativity) in the stellar oscillation equations. Finn (1988) proposed that the McDermott, Van Horn, and Scholl (1983) approximation be refined by retaining the δg_{rt} component of the perturbed metric in the pulsation equations. By considering the Newtonian limit of these equations, he observed that this component of the metric is larger than the other metric components under the circumstances pertinent to the Cowling approximation. In this paper we evaluate the accuracy of these two generalizations of the Cowling approximation to relativistic stellar pulsations. We compute the frequencies of several low-order p-modes using both versions of the approximation and compare the results to the frequencies of the exact relativistic stellar pulsation equations. We also measure the accuracy of the two versions of the Cowling eigenfunctions by comparing the values of the integrals of these functions that determine the viscous energy dissipation rate.

Section II of this paper discusses the mathematical formalism needed to describe the nonradial oscillations of nonrotating general relativistic stellar models in the Cowling approximations of McDermott, Van Horn, and Scholl (1983) and Finn (1988). The approximate versions of the pulsation equations are presented along with a variational principle for the frequencies which are the eigenvalues of these equations. In order to measure the accuracy of the Cowling eigenfunctions, we describe how the viscous dissipation time scale can be computed as a ratio of integrals which are quadratic in the eigenfunctions. The viscous dissipation time scale is therefore a kind of "Sobolev norm," whose value is a relevant quantitive test of the accuracy of the Cowling eigenfunctions. Section III describes the results of our numerical calculations of the frequencies and viscous dissipation times for several of the lower order dipole p-modes of an $n = 1$ polytrope using both versions of the relativistic Cowling pulsation equations. We have chosen to limit our investigation here to the dipole modes in order to be able to investigate the higher order p-modes. The $l \geq 2$ modes couple to gravitational radiation, and the currently available algortihms fail when the imaginary part of the frequency is too small (e.g., in modes with large values of l or in the higher order p-modes). We have also chosen to limit our investigation here to stars having an $n = 1$ polytropic equation of state. This equation of state has overall properties which are very similar to the more realistic descriptions of neutron star matter (so that we expect the results found here to be typical) but polytropes are much smoother than the tabulated realistic equations of state so the numerical analysis is simpler and more reliable. We find that the solutions to the McDermott, Van Horn, and Scholl (1983) equations are far better approximations to the exact pulsations than those based on the Finn (1988) equations for the dipole p-modes studied here.

II. THE RELATIVISTIC COWLING APPROXIMATION

The equations which define the relativistic Cowling approximations are presented in this section. In addition a variational principle for the approximate frequencies and expressions for the approximate viscous dissipation time scale are described.

a) Background Stellar Models

The gravitational field of a static spherical star in general relativity theory is represented by the metric tensor,

$$ds^2 = -e^\nu dt^2 + e^\lambda dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\,, \tag{1}$$

where λ and ν are functions of r only. The functions λ, ν, and the pressure p are determined by Einstein's equation

$$\lambda' = \frac{1}{r}(1 - e^\lambda) + 8\pi r \rho e^\lambda\,, \tag{2}$$

$$\nu' = -\lambda' + 8\pi r e^\lambda(\rho + p)\,, \tag{3}$$

$$p' = -\tfrac{1}{2}(\rho + p)\nu'\,, \tag{4}$$

where prime denotes differentiation with respect to r, ρ is the enrgy density of the fluid and $G = c = 1$. Given an equation of state $\rho = \rho(p)$, the integration of these equations is straightforward and well understood.

b) Nonradial Oscillations in the Cowling Approximation

The adiabatic perturbations of an equilibrium stellar model can be desribed by the Lagrangian displacement vector, ξ^a, and the Eulerian perturbation in the metric tensor, δg_{ab}. McDermott, Van Horn, and Scholl (1983) suggest that an appropriate generalization of the Cowling approximation to relativistic stellar pulsations is to neglect the metric perturbations (i.e., set $\delta g_{ab} = 0$) and use the perturbed conservation laws to determine ξ^a. Finn (1988) argues that it is more appropriate to retain in the pulsation equations the δg_{rt} component of the perturbed metric. In the Newtonian limit this component may be larger than the other compoments of the perturbed metric under the circumstances pertinent to the Cowling approximation (see Thorne 1969). Using the gauge choices of Campolattaro and Thorne (1970) (for $l = 1$) and Thorne and Campolattaro (1967) (for $l \geq 2$) we parameterize the Lagrangian displacement and metric perturbations as follows:

$$\xi_a = \left(\frac{e^{\lambda/2}}{r^2} W Y^l_m \nabla_a r + V \nabla_a Y^l_m\right) e^{i\omega t} , \tag{5}$$

and

$$\delta g_{ab} dx^a dx^b = 2i\omega H_1 Y^l_m e^{i\omega t} dt\, dr , \tag{6}$$

where W, V, and H_1 are functions of r only; Y^l_m is the standard spherical harmonic, ω is the frequency of the mode, and λ is the metric function from the background stellar model (eqs. [1]–[4]). The equations for these quantities follow from the perturbed conservation laws [i.e. $\delta(\nabla_a T^a_b) = 0$] and from the $\delta G^r_t = 8\pi \delta T^r_t$ component of the perturbed Einstein equation. They can be represented as follows,

$$\delta p' = -\frac{1}{2}\left(1 + \frac{\rho + p}{p\gamma}\right)\nu' \delta p + \frac{\omega^2}{r^2} e^{-\nu + \lambda/2}(\rho + p)(W + r^2 e^{-\lambda/2} H_1) , \tag{7}$$

$$W' = -\frac{p'}{p\gamma} W - \frac{r^2 e^{\lambda/2}}{p\gamma}\left(1 - \frac{l(l+1)e^{\nu} p\gamma}{r^2\omega^2(\rho + p)}\right)\delta p , \tag{8}$$

$$H_1 = -\frac{16\pi(\rho + p)e^{\lambda/2}}{l(l+1)} W , \tag{9}$$

$$V = \frac{e^{\nu}\delta p}{\omega^2(\rho + p)} , \tag{10}$$

where δp represents the (Eulerian) perturbation in the pressure. These equations are identical to those given by Finn (1988) in equations (2.17)–(2.20) (up to changes in sign convention and a typographical error in equation [2.17]). In this paper we are interested in determining the accuracy of the Cowling approximation for describing the p-mode oscillations. Consequently, we choose to use the simplest model for the adiabatic index, γ, based on the structure of the equilibrium stellar model

$$\gamma = \frac{\rho + p}{p}\frac{p'}{\rho'} . \tag{11}$$

All of the g-mode frequencies are zero when γ has this value. The relativistic Cowling approximation of McDermott, Van Horn, and Scholl (1983) is also described by the equations presented above when equation (9) is replaced by,

$$H_1 = 0 , \tag{12}$$

thereby setting all metric perturbations to zero.

Boundary conditions must be given for the functions δp and W at the surface ($r = R$) and at the center ($r = 0$) of the star. At the surface we require that the Lagrangian change in the pressure vanish

$$0 = \Delta p(R) = \delta p(R) - \rho(R)\frac{M}{R^4} e^{\lambda(R)/2} W(R) . \tag{13}$$

At the center of the star the boundary conditions are determined by the requirement that the physical perturbation variables remain finite (i.e., ξ^a, δg_{ab}, δp, etc.). Imposing these conditions on the differential equations for the perturbation variables results in the conditions:

$$\lim_{r\to 0}\left(\frac{W}{r^{l+1}}\right) = w , \tag{14}$$

and

$$\lim_{r\to 0}\left(\frac{\delta p}{r^l}\right) = \frac{\omega^2 e^{-\nu_c}}{l}(\rho_c + p_c) w , \tag{15}$$

where w is an arbitrary constant, and ρ_c, p_c, and ν_c are the values of the density, the pressure, and the background metric function, ν, at the center of the star. We point out that the value of $H_1(R)$ implied by equation (9) is not consistent with the boundary value of this quantity given by the exact relativistic pulsation equations for the case $l = 1$ unless $\rho(R) = 0$. This inconsistency should not be too surprising, however, because the full Einstein equation is not imposed on the Cowling eigenfunctions.

c) *A Variational Principle for the Frequencies*

A variational principle for the frequencies of the normal modes of an oscillating system is a useful tool for finding the eigenvalues of the system numerically. We use the variational principle described here to obtain initial estimates of the frequency of the mode under study, and once the equations have been solved, we use the variational principle as a check on the accuracy of the calculation. Detweiler and Ipser (1973) and Detweiler (1975) have derived variational principles for the exact relativistic pulsation equations. Their derivation used the perturbed conservation laws, but not the perturbed Einstein equation. Since the equations for the relativistic Cowling approximation are just the pertubed conservation laws, the exact variational principle is applicable simply by setting the unused components of the perturbed metric to zero: $K = H_0 = H_2 = 0$. Equivalently, the following expression for the frequency of a mode in terms of its eigenfunctions follows directly from equations (7)–(15):

$$\omega^2 \int_0^R e^{-\nu/2} \times \left\{e^{\lambda/2}(\rho + p)\left[\frac{W^2}{r^2} + l(l+1)V^2\right] - \frac{l(l+1)e^{-\lambda/2}}{16\pi} H_1^2\right\} dr = \int_0^R r^2 e^{(\lambda+\nu)/2}\frac{(\delta p)^2}{p\gamma} dr + \left[\frac{\rho(R) M W^2(R)}{R^4}\right] . \tag{16}$$

The metric function H_1 is given here either by equation (9) or by equation (12).

d) *Viscous Dissipation Time Scales*

While it is easy to compare the frequencies predicted by the relativistic Cowling approximation with those based on the exact relativistic pulsation equations, it is less straightforward to estimate the accuracy of the approximate eigenfunctions. We measure this accuracy here by using the eigenfunctions to compute a relevant physical quantity: the viscous dissipation time scale for the mode. This time scale, τ, is given by the ratio of two integrals which are quadratic in the eigenfunctions:

$$\frac{1}{\tau}=\frac{\omega^2}{E}\int \eta r^2 e^{\lambda/2} \times\left\{6\alpha_1^2+2l(l+1)\alpha_2^2+l(l+1)\left[\frac{1}{2}l(l+1)-1\right]\frac{V^2}{r^4}\right\}dr\,, \quad (17)$$

where the energy, E, in the mode is given by the integral,

$$E=\omega^2\int_0^R e^{-\nu/2} \times\left\{e^{\lambda/2}(\rho+p)\left[\frac{W^2}{r^2}+l(l+1)V^2\right]-\frac{l(l+1)e^{-\lambda/2}}{16\pi}H_1^2\right\}dr\,, \quad (18)$$

and α_1 and α_2 are defined as

$$\alpha_1=e^{-\lambda/2}\left(\frac{W'}{3r^2}-\frac{W}{r^3}\right)+\frac{l(l+1)V}{6r^2}\,, \quad (19)$$

$$\alpha_2=e^{-\lambda/2}\left(\frac{V'}{2r}-\frac{V}{r^2}\right)+\frac{W}{2r^3}\,. \quad (20)$$

The dissipation time scale, τ, is therefore a kind of "Sobolev norm" on the eigenfunctions of the stellar pulsation equations. It can be used therefore as a measure of the accuracy of these eigenfunctions when comparing the Cowling approximation to the exact relativistic stellar pulsation equations. The exact expressions for the viscous dissipation time scales are given in Cutler and Lindblom (1987) (up to typographical errors) for the case $l \geq 2$ and by Lindblom and Splinter (1989) for $l = 1$.

III. NUMERICAL SOLUTIONS OF THE RELATIVISTIC COWLING EQUATIONS

In this section we discuss the numerical methods that were used to solve the Cowling-approximation pulsation equations and we present the results of those calculations. Equations (7)–(12) with the boundary conditions equations (13)–(15) comprise a two-point-boundary eigenvalue problem. We solve this system iteratively by integrating the equations from both boundaries then adjusting the boundary values and the frequency until the solutions match smoothly at some interior matching point (see e.g., Lindblom and Splinter 1989). This procedure is relatively straightforward except for imposing the boundary conditions.

At the center of the star ($r = 0$) the boundary conditions are easily implemented by using the power series solutions implicit in equations (14) and (15) to obtain the values of W and δp on the first grid point away from $r = 0$. The situation is more problematical at the surface of the star ($r = R$) because the function W is sharply peaked there. A power series can, nevertheless, be used to obtain the values of W and δp on the grid point nearest the boundary ($r = R$). Since this power series contains terms at each order (unlike the situation at $r = 0$ where every other term vanishes), we find that an expansion beyond the first nontrivial order is needed. The needed expressions for $W(r)$ and $\delta p(r)$ are obtained by expanding equations (7)–(13) in powers of $\epsilon = 1 - r/R$. A knowledge of the power series solution for the background stellar model is needed to perform these expansions. In this paper we will be concerned only with stars having polytropic equations of state ($p = K\rho^{1+1/n}$) for which these power series solutions are easily obtained (see e.g., Thorne and Campolattaro 1967). In this case the power series for W and δp have the form:

$$W = W(R)[1 + \Lambda_W \epsilon + O(\epsilon^2)]\,, \quad (21)$$

$$\delta p = \delta p_0\, \epsilon^n[1 + \Lambda_p \epsilon + O(\epsilon^2)]\,, \quad (22)$$

where

$$\delta p_0=\frac{W(R)}{R^3}\left[\frac{1}{K(1+n)}\left(\frac{M}{R}\right)^{n/(1+n)}\left(1-\frac{2M}{R}\right)^{-(1+2n)/2n}\right]^n, \quad (23)$$

and $W(R)$, Λ_w and Λ_p are constants. The constants Λ_w and Λ_p could be determined analytically by a straightforward but lengthy calculation. We choose to find them instead using a mixed analytic-numerical approach. Differentiating equations (21) and (22) and equating the results to the expressions for the derivatives given in equations (7) and (8) we find

$$-\delta p_0 R^{-1}\epsilon^{n-1}[n+(n+1)\Lambda_p\epsilon] = C_1 W(R)(1+\Lambda_w\epsilon)+C_2\,\delta p_0\,\epsilon^n(1+\Lambda_p\epsilon)\,, \quad (24)$$

$$-W(R)R^{-1}\Lambda_w = C_3 W(R)(1+\Lambda_w\epsilon)+C_4\,\delta p_0\,\epsilon^n(1+\Lambda_p\epsilon)\,, \quad (25)$$

where the coefficients C_i are given implicitly in equations (7) and (8). The values of the C_i and ϵ are determined numerically on the last grid point inside the surface of the star. Equations (24) and (25) are easily solved then for the constants Λ_w and Λ_p which, in turn, determine the values of W and δp on the last grid point via equations (21) and (22).

Our purpose in this paper is to evaluate the accuracy of the two versions of the relativistic Cowling approximation discussed in § II. We take for our background stellar model, therefore, a moderately relativistic star ($M = 1.0\, M_\odot$ and $R = 10.18$ km) based on a polytropic equation of state of index $n = 1$: $p = K\rho^2$ with $K = 6.673 \times 10^4$ in cgs units. While this model is similar in many respects to realistic neutron stars, its equation of state and consequently its overall structure are smoother. Table 1 presents the frequencies of a number of the lower order dipole ($l = 1$) p-modes for this model as computed using the exact relativistic pulsation equations (as described in Lindblom and Splinter 1989) and using the two versions of the relativistic Cowling approximation discussed in § II. We evaluated the numerical accuracy of these frequencies by comparing the directly determined eigenvalue of the equation with the variational expression for the frequency (eq. [16]) using the directly determined eigenfunctions. These two values of the frequencies differ by less than 0.1% for all of the frequencies reported here. We also verified that the difference, between the most accurate value of the frequency determined and the value computed with a model having N grid points, varied approximately like $1/N^2$. This scaling of the error is consistent with the accuracy with which the boundary conditons were imposed. The results presented in Table 1 reveal that the original version of the relativistic Cowling approximation proposed by McDermott, Van Horn, and Scholl (1983) has frequencies which are uniformly more accurate than those based on the refined approximation of Finn (1988). For the modes containing more than about three nodes in the radial eigenfunction, the McDermott, Van Horn, and Scholl (1983) equations are in fact *substantially* more accurate than Finn's (1988) equations at least in the case of the dipole p-modes studied here. The reason for the inaccuracy of Finn's approximation is clearly illustrated in Figure 1. There are depicted the functions H_0, ωH_1, and H_2 which represent the metric perturbations for the p_1-mode as determined by the exact pulsation equations (see Lindblom and Splinter 1989). These functions are normalized so that $W(R) = 1$. These functions clearly violate Finn's assumption that $\delta g_{rt} = i\omega H_1 Y^l_m e^{i\omega t}$ dominates the other components of the perturbed metric. Figure 1 also shows that the function ωH_1 as determined by Finn's equations is a poor approximation of the exact eigenfunction. We note that the stellar model studied here satisfies $\alpha \equiv 1 - 16\pi r^2(\rho + p)/[l(l+1)] > 0.44$ and thus, is free of the instability in the Cowling approximation described by Finn (1988) when $\alpha = 0$. The dispersion relation (Finn's eq. [3.28]) for Finn's equations indicates that the frequencies should be larger than those based on the McDermott, Van Horn, and Scholl equation by a factor of $\alpha^{-1/2}$. This is roughly consistent with the numerical results obstined here, and suggests that the difference between the two approximations is likely to be smaller for larger values of l. In the Newtonian limit the term containing the metric perturbation H_1 in equation (7) becomes negligible compared to the term containing W (as anticipated by Finn 1988). In this limit, the two forms of the relativistic Cowling approximation approach one another. We verified that the frequencies of the two approximations do approach each other in this limit. For a stellar model having $GM/c^2R = 1.4 \times 10^{-4}$ we find that the frequencies of the $l = 1$ p_1-mode in the two versions of the Cowling approximation differ by only 0.01%. It is important to emphasize that we have evaluated the accuracy of the relativistic Cowling approximations only for the $l = 1$ p-modes. We think that it is unlikely that the Finn equations will prove to be more accurate than the McDermott, Van Horn, and Scholl equations for the higher l p-modes; however, the Finn equations may well provide better descriptions of the g-modes for the reasons discussed by Finn (1988).

TABLE 1

FREQUENCIES OF THE DIPOLE p-MODES IN THE COWLING APPROXIMATION[a]

NODES[b]	FREQUENCIES[c]			VISCOUS DAMPING TIMES[d]		
κ	ω_E/ω_κ	$\omega_{MVS}/\omega_\kappa$	ω_F/ω_κ	τ_E/τ_κ	τ_{MVS}/τ_κ	τ_F/τ_κ
1	1.325	1.595	1.809	5.36	4.00	2.95
2	1.512	1.641	1.955	3.06	2.69	1.90
3	1.573	1.648	1.962	2.57	2.39	1.71
4	1.597	1.647	1.950	2.38	2.28	1.62
5	1.609	1.644	1.945	2.29	2.22	1.58
10	1.619	1.630	1.924	2.18	2.16	1.55
15	1.617	1.623	1.914	2.17	2.16	1.55
20	1.615	1.618	1.908	2.17	2.16	1.55

[a] Three frequencies and viscous damping times are given for each mode: E refers to the exact relativistic frequency, MVS to the relativistic Cowling approximation of McDermott, Van Horn and Scholl (1983), and F to the relativistic Cowling approximation of Finn (1988).

[b] The number of nodes in the radial component of the Lagrangian displacement is denoted by κ.

[c] The reference frequency is defined as $\omega_\kappa \equiv (\kappa + 1)\sqrt{\pi G \rho_{avg}}$, where $\rho_{avg} = 3M/4\pi R^3$.

[d] The reference damping time is defined as $\tau_\kappa \equiv R^2 \rho_{avg}/[(\kappa+1)^2 \eta(\rho_{avg})]$, where η is the viscosity.

We have also evaluated the accuracy of the fluid motions predicted by the two versions of the relativistic Cowling approximation. Figure 2 illustrates the radial component of the Lagrangian displacement of the fluid for the p_1-mode as computed using the exact relativistic pulsation equations and the two versions of the relativistic Cowling approximation. While all of these eigenfunctions are qualitatively similar, both versions of the Cowling eigenfunctions are more strongly peaked at the surface of the star than the exact eigenfunctions. This (ironically) makes the approximate equations more difficult to solve numerically than the exact equations (at least in the dipole case studied here where gravitational radiation effects are absent). In order to obtain a quantitative measure of

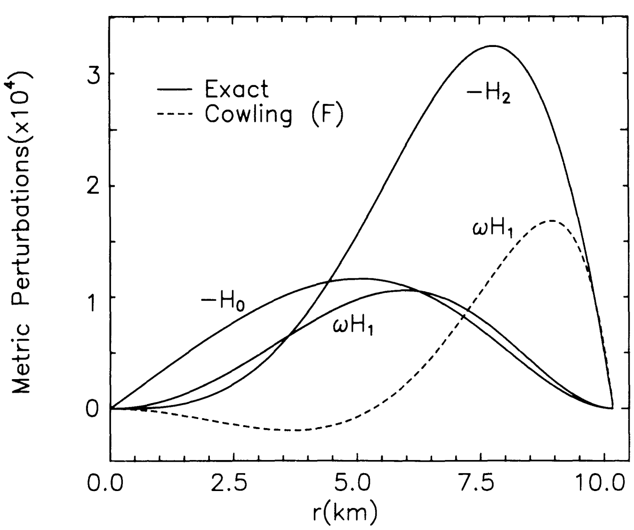

FIG. 1.—The solid curves illustrate the complete set of metric perturbations for the p_1-mode based on the exact pulsation equations. The dashed curve illustrates the only nonvanishing metric perturbation in the Finn version of the Cowling approximation.

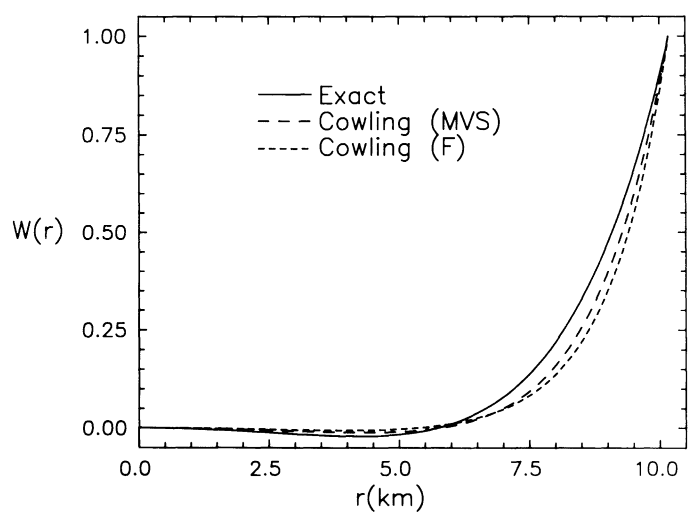

FIG. 2.—The radial component of the Lagrangian displacement, based on the exact pulsation equations and two versions of the Cowling approximation, is illustrated for the p_1-mode. Cowling (MVS) refers to the approximation of McDermott, Van Horn, and Scholl, while Cowling (F) to the approximation of Finn.

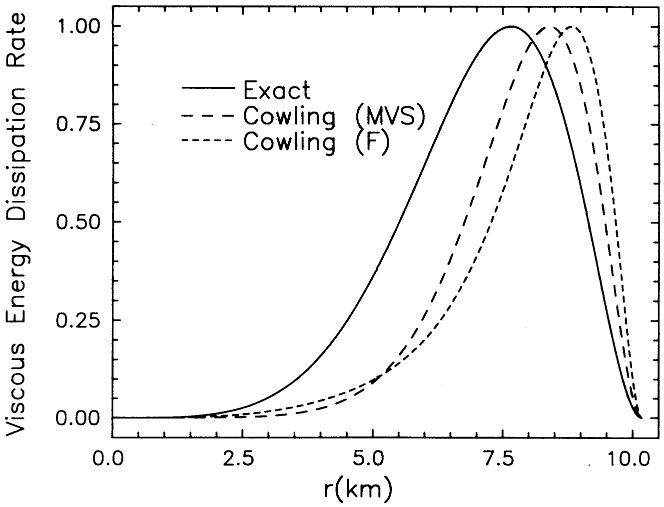

FIG. 3.—The viscous energy dissipation rate (the integrand of eq. [18]), for the eigenfunctions based on the exact pulsations and two versions of the Cowling approximation, is illustrated for the p_1-mode. Cowling (MVS) refers to the approximation of McDermott, Van Horn, and Scholl, while Cowling (F) to the approximation of Finn.

the accuracy of these eigenfunctions we used them to evaluate the viscous dissipation timescale for these modes as described in equations (17)–(20). We used the electron-electron scattering viscosity (see, e.g., Cutler and Lindblom [1987]) given by the formula: $\eta = 6 \times 10^6 \rho^2/T^2$ (in cgs units) which is appropriate for neutron stars cooler than $\sim 10^9$ K. We assume that the star is "isothermal" in the relativistic sense that $Te^{\nu/2}$ is constant. Figure 3 illustrates the integrand in equation (17) for the p_1-mode of the exact and the two approximate sets of eigenfunctions. This figure shows that these modes contain significant fluid motions which are less superficial than would be anticipated by examining the eigenfunctions themselves (i.e., Fig. 2). We note that this integrand continues to be non-negligible deep within the star even for modes having many nodes. Table 1 contains the numerical values of the dissipation time scales for the exact relativistic pulsations and the two approximations. These results reveal the remarkable accuracy of the relativistic Cowling approximation of McDermott, Van Horn, and Scholl (1983) for predicting the frequencies and eigenfunctions for p-modes having more than about five nodes.

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