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THE ROLE OF NEUTRINO DISSIPATION IN GRAVITATIONAL COLLAPSE[1]

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ABSTRACT

The neutrino radiation, trapped in the core of a collapsing star, serves as an efficient mechanism for transporting energy and momentum throughout the core. We show, however, that the associated radiative viscosity is ineffective in damping differential rotation in the stellar core. We also compare the effectiveness of damping the oscillations of a stellar core by neutrino dissipation and by gravitational radiation. We find that gravitational radiation will provide the dominant damping mechanism for the low l modes in sufficiently dense stellar cores: $\rho \gtrsim 10^{13}$. Neutrino dissipation provides the damping for lower density cores, and for modes with higher l values. Our estimates indicate that neutrinos do not dominate the damping of core oscillations to the extent found by Kazanas and Schramm.

Subject headings: gravitation — neutrinos — stars: collapsed — stars: pulsation

I. INTRODUCTION

Recent studies of the collapse of stellar cores show that the neutrino cross sections are large enough that the neutrinos become trapped in the core, and therefore have time to reach thermal equilibrium with the matter (Arnett 1977; Sawyer and Soni 1979). When this situation occurs, it is appropriate to approximate the momentum and energy transport due to neutrinos in the core as a classical radiative viscosity and heat conduction. The coefficients of viscosity η and ζ, and the heat conduction coefficient κ, due to the dissipation from a thermal neutrino gas with energy density ρ_v, are given by Weinberg (1971):

$$\eta = \tfrac{1}{3}\rho_v c\lambda\,, \tag{1}$$

$$\zeta = 4\rho_v c\lambda\left[\frac{1}{3} - \left(\frac{\partial p}{\partial \rho}\right)_n c^{-2}\right]^2, \tag{2}$$

$$\kappa = \frac{1}{3}\frac{\partial \rho_v}{\partial T}\,c\lambda\,, \tag{3}$$

$$\rho_v = \left(\frac{15}{2\pi^4}\right) aT^4 F_3(\xi)\,. \tag{4}$$

The constants a and c are the Stefan-Boltzmann constant and the speed of light, while λ and ξ represent the mean free path and the degeneracy of the neutrinos, and F_3 is a Fermi integral. We compute here the effects of these dissipation terms on the dynamics of an ideal fluid of neutrons, representing the matter in the stellar core. The characteristic time scales for the cooling of the stellar core, τ_c, the damping of differential rotation due to viscous dissipation, τ_v, the damping of acoustical waves, τ_A, and the damping of the nonradial modes by gravitational radiation, τ_G, are computed. We show that the ratio τ_v/τ_c is always greater than one; consequently, the neutrinos are not effective in damping out differential rotation, despite the large numerical values of the viscosity coefficients (see Kazanas 1978). Furthermore, we show that the ratio τ_A/τ_c is less than one, unless the temperature of the core has fallen significantly below the Fermi energy of the neutron gas in the core. Thus the neutrinos can be effective in damping the oscillations of the stellar core while the core is hot. Finally, we compare the damping of the nonradial modes of the core due to neutrino dissipation and gravitational radiation. We find that the emission of gravitational radiation is less efficient than the neutrinos in dissipating the oscillation energy in stellar cores, except for the modes having small spherical harmonic index l in sufficiently dense stellar cores; $\rho \gtrsim 10^{13}$. Our method of treating the neutrino dissipation and our conclusions differ from those of Kazanas and Schramm (1977) and Kazanas (1978).

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II. COMPUTING THE DISSIPATION TIME SCALES

The damping of differential rotation in a rotating stellar core is governed by the following terms from the Navier-Stokes equation (see Landau and Lifshitz 1959, § 15):

$$\rho\frac{\partial v^i}{\partial t} = \nabla_j[\eta(\nabla^i v^j + \nabla^j v^i - \tfrac{2}{3}\delta^{ij}\nabla_k v^k)]\,. \tag{5}$$

The time scale over which the viscosity is effective in changing the velocity distribution in the core can be estimated by considering the dimensional form of equation (5):

$$\rho\frac{R\Omega}{\tau_v} = \frac{\eta\Delta\Omega}{R}\,. \tag{6}$$

In this equation ρ represents the average density of the core, η the average viscosity, R the radius of the core, Ω the average angular velocity, $\Delta\Omega$ the average spatial variation of the angular velocity from the mean value, and τ_v the viscous damping time scale. From this equation, the standard expression for the viscous damping time (the time required to damp velocity variations of the order $\Delta\Omega \sim \Omega$) can be inferred:

$$\tau_v = \frac{\rho R^2}{\eta} . \tag{7}$$

The cooling of the stellar core is determined by the heat diffusion equation (see Landau and Lifshitz 1959, § 50)

$$\rho c_p \frac{\partial T}{\partial t} = m\nabla^i(\kappa\nabla_i T) . \tag{8}$$

In this equation c_p is the specific heat (per particle) at constant pressure, κ the heat conduction coefficient, T the temperature of the fluid, and m the mass of a particle in the fluid (the neutron mass for our problem). The surface of the core is kept at a very low temperature due to the radiation of neutrinos; consequently, the temperature variation in the core is always on the order of the average value of the temperature: $\Delta T \sim T$. Therefore the dimensional form of equation (8) is given by

$$\rho c_p \frac{T}{\tau_c} = \frac{m\kappa T}{R^2} , \tag{9}$$

where τ_c is the characteristic cooling time of the core. The cooling time scale is thus given by the expression

$$\tau_c = \frac{\rho R^2 c_p}{m\kappa} . \tag{10}$$

The specific heat of the stellar core is due primarily to the neutron gas in the core. If the temperature of the core is cool enough that the neutrons are degenerate, $kT/\epsilon_F \ll 1$ (ϵ_F is the Fermi energy of the neutron gas), then the specific heat is given by the expression (Huang 1963)

$$c_p = \frac{\pi^2}{2} k \left(\frac{kT}{\epsilon_F}\right) . \tag{11}$$

If the temperature is much hotter, $kT/\epsilon_F \gg 1$, then the neutron gas behaves like a classical Boltzmann gas, and the specific heat is given by the expression

$$c_p = \tfrac{5}{2}k . \tag{12}$$

We compute the values of the various dissipation time scales for both limiting models of the neutron gas. A realistic collapse results in temperatures intermediate between these limiting cases, $kT/\epsilon_F \sim 1$, which then cool toward the degenerate Fermi gas limit.

We estimate the damping of acoustical waves in the stellar core by treating the neutron gas as isothermal and uniform-density. This model is not absolutely appropriate for describing stellar cores, which are qualitatively isothermal, but not uniform-density (see Arnett 1977). However, this model is probably better than the other model which yields simple analytic results (a uniform-density self-gravitating gas which is not isothermal), since the damping process is primarily a thermodynamic process, and the thermodynamic functions, T and ρ, in the completely uniform case more closely resemble the situation in the real stellar core.

The characteristic damping time of acoustical waves in a uniform-density isothermal medium is computed in Landau and Lifshitz (1959, § 77):

$$\tau_A = 2\rho \frac{v_s^2}{\omega^2}\left[\frac{4}{3}\eta + \zeta + m\kappa\left(\frac{1}{c_v} - \frac{1}{c_p}\right)\right]^{-1} . \tag{13}$$

In this equation v_s is the velocity of sound in the material, and ω is the frequency of the acoustical wave. The specific calculation of the damping time presented in Landau and Lifshitz (1959) is for the damping of plane waves, but the same expression describes the damping of spherical waves also (see Rayleigh 1878, § 349). Since the expression is quite independent of the geometry of the waves, we feel it is possible to apply it here to the damping of the acoustical oscillations of a stellar core.

The needed combination of specific heats for the degenerate Fermi gas limit is given by the expression

$$\frac{1}{c_v} - \frac{1}{c_p} = \frac{2}{3}\frac{T}{\epsilon_F} , \tag{14}$$

while the expression in the case of a Boltzmann gas has the form

$$\frac{1}{c_v} - \frac{1}{c_p} = \frac{4}{15k} . \tag{15}$$

With these expressions for the specific heats, and the expressions for the heat conduction and viscosity coefficients from equations (1)–(3), we find that the viscosity contribution to the acoustical damping in equation (13) is negligible in the stellar core compared with the contribution from the heat conduction. The heat conduction contribution is larger by a factor of mc^2/ϵ_F (mc^2/kT) for the degenerate Fermi gas (Boltzmann gas) case. We therefore neglect the viscosity contributions, and compute the acoustical damping by the formula

$$\tau_A = \frac{2\rho v_s^2}{m\kappa\omega^2}\left(\frac{1}{c_v} - \frac{1}{c_p}\right)^{-1} . \tag{16}$$

The speed of sound for the neutron gas is computed from the formula

$$v_s^2 = \frac{2\epsilon_F}{3m} \tag{17}$$

for a degenerate Fermi gas and

$$v_s^2 = \frac{5kT}{3m} \tag{18}$$

for a Boltzmann gas. For the frequency of the sound wave which appears in equation (16), we use the formula for the frequencies of the Kelvin modes of nonradial oscillation of an incompressible self-gravitating gas sphere (see Chandrasekhar 1961, § 98):

$$\omega^2 = \frac{2GM}{R^3}\frac{l(l-1)}{2l+1}, \tag{19}$$

where G is the gravitation constant, M is the mass of the core, and l is the index of the spherical harmonic which describes the mode.

The final dissipation time scale to be considered here is that for the damping of the nonradial modes by the emission of gravitational radiation. This time scale has been computed for the oscillations of an incompressible self-gravitating sphere by Detweiler (1975), who finds the formula,

$$\tau_G = \frac{4l(l-1)^2(2l+1)[(2l-1)!!]^2c^{2l+1}}{3(l+1)(l+2)\omega^{2l+2}R^{2l+1}}. \tag{20}$$

The frequency of oscillation of a particular mode is found from equation (19).

To summarize our estimates of the various dissipation time scales, we list their formulae here. In these formulae we have included the expressions for the thermal properties of the neutron gas, and the expression for the frequencies of the normal modes. The formulae presented here are those which are appropriate when the neutrino degeneracy is zero: $\xi = 0$. For degenerate neutrinos the dissipation time scales τ_v, τ_c, and τ_A will be smaller by the factor $F_3(0)/F_3(\xi) < 1$.

$$\tau_v = \frac{45Mc}{14\pi aT^4\lambda R}, \tag{21}$$

$$\tau_c(\text{Fermi}) = \frac{9\pi Mk}{28mcaT^3\lambda R}\left(\frac{kT}{\epsilon_F}\right), \tag{22}$$

$$\tau_c(\text{Boltzmann}) = \frac{45Mk}{28\pi mcaT^3\lambda R}, \tag{23}$$

$$\tau_A(\text{Fermi}) = \frac{9(2l+1)k^2}{14\pi l(l-1)Gm^2caT^2\lambda}\left(\frac{\epsilon_F}{kT}\right)^2, \tag{24}$$

$$\tau_A(\text{Boltzmann}) = \frac{225(2l+1)k^2}{56\pi l(l-1)Gm^2caT^2\lambda}, \tag{25}$$

$$\tau_G = \frac{2^{1-l}(2l+1)^{l+2}[(2l-1)!!]^2}{3l^l(l-1)^{l-1}(l+1)(l+2)}\left(\frac{c^2R}{GM}\right)^{l+1}\frac{R}{c}. \tag{26}$$

III. DISCUSSION

To determine whether the neutrinos are effective in damping out differential rotation in the core, one must compare the viscous damping time scale with the cooling time of the core. After the core cools to about 10^9 K, the neutrino mean free paths become much larger than the size of the core. Consequently, the neutrinos cease to provide an efficient mechanism for the redistribution of energy and momentum within the core. Therefore only if the damping time scale, τ_v, is shorter than the cooling time scale, τ_c, will the neutrinos be present in the core long enough to effectively damp out differential rotation.

Using equations (21)–(23), we compute the ratio τ_v/τ_c:

$$\frac{\tau_v}{\tau_c}(\text{Fermi}) = \frac{10}{\pi^2}\left(\frac{mc^2}{kT}\right)\frac{\epsilon_F}{kT} \gg 1, \tag{27}$$

$$\frac{\tau_v}{\tau_c}(\text{Boltzmann}) = \frac{mc^2}{kT} \gg 1. \tag{28}$$

Since the ratio τ_v/τ_c is much larger than one for the typical parameters of a stellar core, it follows that neutrino viscosity is never effective in damping out differential rotation in the stellar core unless the angular velocity variation is very large indeed: $\Delta\Omega/\Omega \gtrsim 2mc^2/kT \gtrsim 100$. We note that this conclusion is essentially independent of the size of the neutrino cross sections, since the mean free path, λ, does not appear in the ratio τ_v/τ_c, and is also independent of the value of the neutrino degeneracy. Our calculation does, however, assume that the scattering cross sections are large enough that the neutrinos remain in thermal equilibrium with the matter in the core.

We consider second the damping of acoustical waves by neutrino dissipation. Using equations (22)–(25) we compute the ratios τ_A/τ_c:

$$\frac{\tau_A}{\tau_c}(\text{Fermi}) = \frac{2}{\pi^2}\frac{2l+1}{l(l-1)} \times \left(\frac{3M\epsilon_F}{5m}\right)\left(\frac{3GM^2}{5R}\right)^{-1}\left(\frac{\epsilon_F}{kT}\right)^2, \tag{29}$$

$$\frac{\tau_A}{\tau_c}(\text{Boltzmann}) = \frac{2l+1}{l(l-1)} \times \left(\frac{3MkT}{2m}\right)\left(\frac{3GM^2}{5R}\right)^{-1}. \tag{30}$$

Note that each of these expressions contains the ratio of the internal energy of the neutron gas ($3M\epsilon_F/5m$ for the Fermi gas and $3MkT/2m$ for the Boltzmann gas) to the gravitational potential energy of the core, $3GM^2/5R$. This ratio will be less than one; consequently, the following inequalities must be satisfied:

$$\frac{\tau_A}{\tau_c}(\text{Fermi}) \lesssim \frac{2}{\pi^2}\frac{2l+1}{l(l-1)}\left(\frac{\epsilon_F}{kT}\right)^2, \tag{31}$$

$$\frac{\tau_A}{\tau_c}(\text{Boltzmann}) \lesssim \frac{2l+1}{l(l-1)}. \tag{32}$$

The acoustical damping time is shorter than the cooling time in these expressions, therefore, unless the star is sufficiently cool, $kT \ll \epsilon_F$. From these expressions it is clear that the neutrinos can be effective in damping the acoustical oscillations before the stellar core cools.

We complete our discussion of the role of neutrino dissipation in gravitational collapse by comparing the effectiveness of neutrino damping of the nonradial modes to the effectiveness of damping by gravitational radiation. The ratio τ_A/τ_G depends crucially on the relative strengths of the gravitational and weak inter-

TABLE 1

Dissipation Time Scales for 1.4 $M_\odot$ Stellar Cores[a]

ρ	T	λ	R	τ_v	τ_c	τ_A	τ_G	$2\pi/\omega$
10^{12}..........	3.4×10^{10}	1.7×10^{5}	8.7×10^{6}	5.7×10^{3}	8.9	7.3	420	7.5×10^{-3}
10^{13}..........	7.2×10^{10}	1.5×10^{4}	4.1×10^{6}	6.8×10^{3}	23	19	21	2.4×10^{-3}
10^{14}..........	1.4×10^{11}	2.5×10^{3}	1.9×10^{6}	6.2×10^{3}	40	29	0.96	7.5×10^{-4}
10^{15}..........	1.8×10^{11}	4×10^{2}	8.7×10^{5}	3.1×10^{4}	260	110	0.042	2.4×10^{-4}

[a] All quantities are presented in cgs units.

actions. Therefore we must explore numerically the values of this ratio for specific core models. We consider several 1.4 $M_\odot$ stellar cores having a wide range of different average densities and temperatures. The average temperatures, densities, and neutrino mean free paths were selected to coincide with the models considered by Kazanas and Schramm (1977). In Table 1 are presented the relevant parameters of these core models, along with the values of the various dissipation time scales computed from equations (21), (23), (25), and (26). The time scales τ_A and τ_G are presented for the $l = 2$ modes. The data from Table 1 indicate that gravitational radiation will provide the dominant mechanism for dissipating the energy in the lowest nonradial modes of oscillation, whenever the stellar core is sufficiently dense: $\rho \gtrsim 10^{13}$. The neutrinos will provide the dissipation for lower density cores, and for the modes with higher values of l.

The numbers presented here are based on the simplified core models described above. For more realistic models, one could expect changes in these time scales due to the following effects. First, neutrino degeneracy will tend to decrease the time scales associated with the neutrino dissipation effects. For a neutrino degeneracy of $\xi = 5$ (see, e.g., Arnett 1977) the time scales τ_v, τ_c, and τ_A would be smaller than the values given in Table 1 by a factor of about 50. Second, the fluid in a realistic stellar core will be a complex mixture of heavy nuclei and free neutrons. The specific heat of such a mixture probably does not differ significantly, however, from the value for a free neutron gas used in our simple models. The nuclei have a large number of possible excited states, which makes the specific heat per nuclei greater than the specific heat per neutron; nevertheless, the specific heat per baryon is probably comparable in the two cases. We thus suspect that a more realistic fluid model will not significantly change the values of time scales in this way. Third, we point out that it is difficult to estimate the differences between realistic gravitational radiation damping times, and those computed here for an incompressible fluid. As an alternative simplistic model, we have computed directly the damping times for the modes of a uniform density, but compressible fluid ($\Gamma_1 = 5/3$) using formulae from Detweiler (1975) and Ledoux and Walraven (1958). We find that the damping times for this model are smaller than those given in Table 1 by a factor of about 15. This effect would suggest that gravitational radiation damping could play the dominant role at densities down to $\rho = 10^{12}$.

Since our methods and conclusions differ from those of Kazanas and Schramm (1977) and Kazanas (1978), it is appropriate to comment on those differences here. Kazanas and Schramm (1977) compute the dissipation of energy in the nonradial modes of a uniform density (but not isothermal) self-gravitating sphere due to neutrino and gravitational radiation damping. They treat the neutrino dissipation by coupling an effective neutrino luminosity function to the energy conservation equation for the neutron fluid. This approach is appropriate whenever the neutrinos have mean free paths longer than the size of the stellar core. The use of the effective neutrino luminosity does not include the effects of neutrino reabsorption by the core, and consequently the approach is not appropriate when the neutrinos are trapped. Kazanas and Schramm (1977) attempt to correct for the reabsorption of neutrinos by multiplying their computed damping times by R/λ, the ratio of the time it takes a neutrino to diffuse out of the core to the time it would take a noninteracting neutrino to escape. Saenz and Shapiro (1978) have argued that the neutrino luminosity calculation of the cooling of the core can be made consistent with the thermal diffusion approximation (as used in this *Letter*) by correcting the neutrino luminosity by the factor $(R/\lambda)^2$. When we correct the damping times computed by Kazanas and Schramm (1977) by this factor, we find damping times which are longer by a factor of 10 than those computed by our equation (23). We interpret this result as meaning that the bulk damping of the core oscillation described by equation (23) is more efficient than the damping by radiating neutrinos from the surface of the core. Therefore the neutrino damping is dominated by the effect described by equation (23). Kazanas (1978) also discussed the damping of acoustical waves in the core, using a formula which ultimately was derived by Weinberg (1971) from equation (13). In the physical situation considered by Weinberg, however, the primary source of heat capacity in the fluid was that of the neutrinos themselves, while we find that the primary source in the stellar core comes from the neutrons. Therefore Kazanas's formula for the damping of acoustical waves is not appropriate.

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