Ph 225b: General Relativity 1 March 2016

WEEK 9: Gravitational Waves

Recommended Reading: R.M. Wald, *General Relativity* (1984), Section 7.5,
C.W. Misner, K.S. Thorne, J.A. Wheeler *Gravitation* (1973),
Sections 35.13-15, 36.1-11

#1 Let the spacetime metric g_{ab} be written in the form $g_{ab} = g^{(B)}_{ab} + h_{ab}$ where $g^{(B)}_{ab}$ is a fixed background metric tensor. Show that the Ricci tensor R_{ab} of g_{ab} is given by

$$R_{ab} = R^{(B)}_{ab} - \frac{1}{2} g^{cd}_{(B)} \Big(\nabla_c \nabla_d h_{ab} + \nabla_a \nabla_b h_{cd} - \nabla_c \nabla_a h_{bd} - \nabla_c \nabla_b h_{ad} \Big) + \mathcal{O}(h^2),$$

where ∇_a is the covariant derivative compatible with $g^{(B)}_{ab}$.

#2 Show that the first-order metric perturbations of flat spacetime h_{ab} and $h_{ab} + \nabla_a v_b + \nabla_b v_a$ represent the same physical spacetime by showing that they have the same first-order Riemann curvature tensor for arbitrary vectors v_a.

#3 Show that one may always choose the gauge vector v_a for a vacuum perturbation so that arbitrary metric perturbations h_{ab} of a spacetime with background metric $g^{(B)}_{ab}$ satisfies the condition $g^{cd}_{(B)} \nabla_c \bar{h}_{da} = 0$ where $\bar{h}_{ab} = h_{ab} - \frac{1}{2} g^{(B)}_{ab} g^{cd}_{(B)} h_{cd}$.

#4 Show that the gravitational field from a weak field slow moving source is given approximately by

$$\bar{h}_{ij} = \frac{2}{r} \frac{d^2 I_{ij}(t-r)}{dt^2},$$

where I_{ij} is the moment of inertia tensor

$$I_{ij}(t) = \int T^{tt}(t, x') x_i x_j d^3 x'.$$

#5 Compute the far-field gravitational perturbation $\bar{h}_{ij}$ generated by a binary star system in a circular orbit with masses m_1 and m_2 and orbital separation R. Assume the stars are far enough apart, $m_1/R << 1$ and $m_2/R << 1$, so that Newtonian dynamics describes the motions of this system.