Lecture 9 Review
There will be assigned seats! On TED grace center on Wed AM

1. What does \( I DTI / d u d u \) represent?

\[
\iint_R f(x, y) \, dx \, dy \quad \text{bit of area}
\]

\[
\iiint f(x, y, z) \, dV = \text{volume}
\]

\[
\iiint f(x, y) \, dA = \text{real area}
\]

But we need real area. What is it?

\( I DTI / d u d u \) represents the stretching factor for area/volume.

So \( d u d u \) is correct area, \( I DTI / d u d u \) corrects it to make it real area.

2. Why \( \iint_R f \, dA = \int_a^b \int_c^d f \, dx \, dy \)?

Cavalieri's: \( \int_a^d \frac{\text{Area}(y)}{\text{volume of slice}} \, dy \)

\( \text{Area}(y) = \int_a^d f \, dx \).

That's informal. Need Fubini's Theorem: So same, if \( f \) is continuous, for instance.

There's a more general version.

3. \( P \) be proportion of whites. Why does \( \iint_{\text{CA}} P \, dA \) and \( \iiint_{\text{CA}} P \, dA \) not make sense?

What are units?

For first, proportion of whites \( \times \text{km}^2 \) or something. Uh... what does that mean?

For second, just proportion, so that seems reasonable.

But... Never add up, or average proportions.

\( \frac{\text{Area} = 1}{\text{Area} = 1} \)

ex

\[
\begin{array}{c|c|c|c}
\text{people} & \text{white} & \text{non-white} \\
\hline
10 & 1 & 9 \quad \text{10%} \\
90 & 9 & 81 \quad \text{90%} \\
\end{array}
\]

\( \iint \frac{P}{dA} = .1 + .9 = 1 \), Area=2, so

\( \frac{1}{\text{CA}} \iint P \, dA = \frac{1}{2} \), so 50% white!

But... Actually, \( \frac{91}{110} = 83\% \) white...

Moral: Adding up works for lots of unusual things, but... never add up average percentages or proportions!
4. \[ \iiint_R \log(x^2+y^2+z^2) \, dV \quad \text{over} \quad R = \Omega \]

region and function are spherical, so use spherical coords.

\[ \iiint_0^2 \int_0^\pi \int_0^{2\pi} \log(r^2) \, r^2 \sin \phi \, dr \, d\phi \, d\theta \]

5. Integrate \( z \, e^{x^2+y^2} \) inside catenoid.

Trick: use cylindrical coords! (Could use spherical, but this is easier)

In cylindrical, \( 0 < \theta < 2\pi, -1 \leq z \leq 1, 0 \leq r \leq \cosh \theta \).

Note, not coordinate box, so have to be careful! (of ordering)

\[ \iiint_{-1}^1 \int_0^{2\pi} \int_0^\cosh(\theta) \, r \, e^r \, r \, dr \, d\theta \, dz \]

6. What is MVT? Why is it obvious? Connected

MVT says that if \( f \) is continuous on \( \Omega \) domain, then it achieves its average value, i.e.,

\[ \frac{1}{\text{area}(R)} \iint_R f \, dA = f(x_0, y_0) \quad \text{for some point} \quad (x_0, y_0) \quad \text{in} \quad R. \]

(or higher dim version)

Why true? Continuous means no breaks or jumps. The avg value is clearly between the highest and lowest values, so the function has to go from high to low. It’s continuous so it can’t “jump” over the average value.