Lecture 8 (6.2, part 2)

Math 20E
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Suppose $T : Q_1 \rightarrow Q_1$ is a map defining coordinates for the 1st quadrant of the plane $Q_1$, by $T(u, v) = (u^2, v^2)$. What is $dA$ in terms of the new coordinates? [Why are these “good” coordinates, when a very similar function did not give “good” coordinates during an earlier lecture?]

(a) $dA = 4uv \, dudv$
(b) $dA = dudv$
(c) $dA = (2u + 2v) \, dudv$
(d) $dA = u^2v^2 \, dudv$
(e) None of these are correct.
Which of these is equivalent to $\int_0^4 \int_1^9 e^{\sqrt{xy}} dx \, dy$?

(a) $\int_0^4 \int_1^9 e^{uv} 4uv \, du \, dv$

(b) $\int_0^2 \int_1^3 e^{uv} 4uv \, du \, dv$

(c) $\int_0^2 \int_1^3 e^{\sqrt{uv}} 4uv \, du \, dv$

(d) $\int_0^2 \int_1^3 e^{uv} \, du \, dv$

(e) None of these are correct.
Which of these is equivalent to \( \iiint_R e^{x^2+y^2} \, dV \), where \( R \) is the cylinder \( x^2 + y^2 \leq 1, \ -1 \leq z \leq 1 \)?

(a) \[ \int_{-1}^{1} \int_{0}^{2\pi} \int_{0}^{1} re^{r^2} \, dr \, d\theta \, dz \]

(b) \[ \int_{-1}^{1} \int_{0}^{2\pi} \int_{0}^{1} e^{r^2} \, r \, d\theta \, dr \, dz \]

(c) \[ \int_{-1}^{1} \int_{0}^{\pi} \int_{0}^{1} re^{r^2} \, dr \, d\theta \, dz \]

(d) \[ \int_{0}^{1} \int_{0}^{2\pi} \int_{-1}^{1} e^{r^2} \, dz \, d\theta \, dr \]

(e) None of these are correct.
The function for spherical coordinates is

\[ T(r, \theta, \phi) = (r \cos \theta \sin \phi, r \sin \theta \sin \phi, r \cos \phi). \]

Set up the matrix in order to find the Jacobian, i.e., to calculate what \( dV \) is in spherical coordinates.
5.

Suppose you want to know the mass of the Earth’s atmosphere. You might assume that the atmosphere is roughly spherically symmetric. If \( \rho \) is the density of the atmosphere at any location (in kg/m\(^3\)), write an integral giving the mass of the atmosphere. (The radius of Earth is about 6371 km, and the atmosphere is essentially gone at 100 km altitude.)

(a) \[ \int_0^{\pi} \int_0^{2\pi} \int_{6371}^{6471} \rho(r) r^2 \sin(\phi) dr d\theta d\phi \]

(b) \[ \int_0^{\pi} \int_0^{2\pi} \int_{6371}^{100} 10^9 \rho(r) r^2 \sin(\phi) dr d\theta d\phi \]

(c) \[ \int_0^{\pi} \int_0^{2\pi} \int_{6371}^{6471} \rho^2 \sin(\phi) dr d\theta d\phi \]

(d) \[ \int_0^{\pi} \int_0^{2\pi} \int_{6371}^{6471} 10^9 \rho(r) r^2 \sin(\phi) dr d\theta d\phi \]

(e) None of these are correct.