Lecture 4 (5.5)

Math 20E
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Let $\rho(x, y, z)$ be the density (in g/cm$^3$) of a ball $B$ at point $(x, y, z)$ (in cm). What is the units of $\iiint_B \rho \, dV$. Explain to your neighbor what the integral represents.

(a) g
(b) g/cm
(c) cm$^3$
(d) g/cm$^3$
(e) None of these are correct.
Let $T(x, y, z)$ be the temperature (in °C) of a ball $B$ at a point $(x, y, z)$ in cm. What is the unit of $\frac{1}{\text{vol}(B)} \iiint_B T \, dV$? What does it represent?

(a) °C cm³
(b) °C/sec
(c) °C
(d) cm³
(e) None of these are correct.
For \( R = [0, 1] \times [-1, 0] \times [-1, 1] \), which of these equals \( \iiint_R f dV \), assuming that \( f \) is integrable?

(a) \( \int_{-1}^{1} \int_{0}^{1} \int_{-1}^{0} f \, dx \, dy \, dz \)

(b) \( \int_{-1}^{1} \int_{0}^{1} \int_{-1}^{0} f \, dz \, dy \, dx \)

(c) \( \int_{-1}^{1} \int_{0}^{1} \int_{-1}^{0} f \, dy \, dz \, dx \)

(d) \( \int_{-1}^{1} \int_{0}^{1} \int_{-1}^{0} f \, dx \, dz \, dy \)

(e) None of these are correct.
If $R$ is the unit sphere, i.e., $x^2 + y^2 + z^2 \leq 1$, which of these equals $\iiint_{R} f \,dV$, assuming that $f$ is integrable?

(a) $\int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} f \,dy\,dx\,dz$

(b) $\int_{-\sqrt{1-x^2-z^2}}^{\sqrt{1-x^2-z^2}} \int_{-\sqrt{1-z^2-y^2}}^{\sqrt{1-z^2-y^2}} \int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} f \,dy\,dx\,dz$

(c) $\int_{-1}^{1} \int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} \int_{-\sqrt{1-x^2-z^2}}^{\sqrt{1-x^2-z^2}} f \,dy\,dx\,dz$

(d) $\int_{-1}^{1} \int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f \,dy\,dx\,dz$

(e) None of these are correct.