Lecture 15 (7.4)

Math 20E
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Explain to your neighbor why, for a parameterized surface, \[ dA = \| T_u \times T_v \| \, du \, dv. \]
Find a parameterization for the vertical cylinder of radius 2.
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(a) \((\theta, z) \mapsto (2 \cos \theta, 2 \sin \theta, z)\)
(b) \((r, \theta, z) \mapsto (r \cos \theta, r \sin \theta, z)\)
(c) \((u, v) \mapsto (u, v, 2 \cos u + 2 \sin v)\)
(d) \((\theta, \phi) \mapsto (2 \cos \theta \sin \phi, 2 \sin \theta \sin \phi, 2 \cos \phi)\)
(e) None of these correct.
3.

Find $dA$ for that vertical cylinder of radius 2, parameterized by

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(a) $d\theta dz$
(b) $2d\theta dz$
(c) $2 \sin \theta \cos \theta d\theta dz$
(d) $\sqrt{\left(\frac{\partial x}{\partial \theta}\right)^2 + \left(\frac{\partial y}{\partial z}\right)^2} d\theta dz$
(e) None of these are correct.
For the graph of $f(x, y)$ parameterized as $F(u, v) = (u, v, f(u, v))$, find $dA$. 

(a) $\frac{\partial f}{\partial u} \frac{\partial u}{\partial v} + \frac{\partial f}{\partial v} \frac{\partial v}{\partial u} \frac{du}{dv}$ 

(b) $\frac{\partial f}{\partial u} \cdot \frac{\partial f}{\partial v} \frac{du}{dv}$ 

(c) $\sqrt{\left(\frac{\partial f}{\partial u}\right)^2 + \left(\frac{\partial f}{\partial v}\right)^2} \frac{du}{dv}$ 

(d) None of these are correct.
For the graph of $f(x, y)$ parameterized as $F(u, v) = (u, v, f(u, v))$, find $dA$.

(a) $dudv$

(b) $\left( \frac{\partial f}{\partial u} + \frac{\partial f}{\partial v} \right) dudv$

(c) $\frac{\partial f}{\partial u} \cdot \frac{\partial f}{\partial v} dudv$

(d) $\sqrt{\left( \frac{\partial f}{\partial u} \right)^2 + \left( \frac{\partial f}{\partial v} \right)^2} dudv$

(e) None of these are correct.
Find a parameterization for the surface of revolution found by taking the graph $y = f(x)$, and rotating it around the $x$-axis.

(a) $(x, y) \mapsto (x, y, f(x))$

(b) $(z, \theta) \mapsto (f(z) \cos \theta, f(z) \sin \theta, z)$

(c) $(x, \theta) \mapsto (x, f(x) \cos \theta, f(x) \sin \theta)$

(d) $(\theta, \phi) \mapsto (f(\cos \theta) \sin (\phi), f(\sin \theta) \sin (\phi), \cos (\phi))$

(e) None of these are correct.
Find a parameterization for the surface of revolution found by taking the graph \( y = f(x) \), and rotating it around the \( x \)-axis.

(a) \( (x, y) \mapsto (x, y, f(x)) \)

(b) \( (z, \theta) \mapsto (f(z) \cos \theta, f(z) \sin \theta, z) \)

(c) \( (x, \theta) \mapsto (x, f(x) \cos \theta, f(x) \sin \theta) \)

(d) \( (\theta, \phi) \mapsto (f(\cos \theta) \sin(\phi), f(\sin \theta) \sin(\phi), \cos(\phi)) \)

(e) None of these are correct.
The surface of revolution found by taking the graph $y = f(x)$, and rotating it around the $x$-axis can be parameterized by $(x, \theta) \mapsto (x, f(x) \cos \theta, f(x) \sin \theta)$. Find $dA$. 

(a) $\int x \, dx \, d\theta$

(b) $\int \sqrt{f'(x)^2 + 1} \, dx \, d\theta$

(c) $\int |f| \sqrt{f'(x)^2 + 1} \, dx \, d\theta$

(d) $\int |f| \sqrt{f'(x)^2 + 1} \cos \theta \, dx \, d\theta$

(e) None of these are correct.
The surface of revolution found by taking the graph \( y = f(x) \), and rotating it around the \( x \)-axis can be parameterized by 
\[(x, \theta) \mapsto (x, f(x) \cos \theta, f(x) \sin \theta)\]. Find \( dA \).

(a) \( dx \, d\theta \)

(b) \( \sqrt{f'^2 + 1} \, dx \, d\theta \)

(c) \( |f| \sqrt{f'^2 + 1} \, dx \, d\theta \)

(d) \( |f| \sqrt{f'^2 + 1} \cos \theta \, dx \, d\theta \)

(e) None of these are correct.