First, do these problems. These will be graded for completion. You must show enough work to convince the grader that you didn’t just copy the answer.

- 8.2: 2, 3 (i.e., do both integrals and show they’re equal.), 5, 9 (7.6.9 was annoying enough without Stokes’ theorem that I dropped it from the homework...), 12, 34

Math majors should do 8.2:26 as well, but don’t turn it in.

Then do these problems. These will be graded for correctness.

1. One bizarre consequence of Stokes’ theorem is that if $S_1$, $S_2$ are two oriented surfaces in $\mathbb{R}^3$ that share the same boundary $\partial S$ with the same induced orientation on $\partial S$, then

$$
\int_{S_1} \text{curl} \mathbf{F} \cdot d\mathbf{S} = \int_{S_2} \text{curl} \mathbf{F} \cdot d\mathbf{S}
$$

for ANY ($C^1$) vector field $\mathbf{F}$.

First, explain how to check that $S_1$ and $S_2$ induce the same orientation on $\partial S$.

Second, explain why these two integrals have to be the same in that case.

Third, how does this change if the induced orientations are not the same?

2. Another bizarre consequence of Stokes’ theorem is that if $S$ is ANY closed surface (i.e., one that closes in on itself, like [the surface of] a sphere), then for ANY $C^1$ vector field $\mathbf{F}$, $\int_S \text{curl} \mathbf{F} \cdot d\mathbf{S} = 0$. Explain why.

3. Use the idea of curl as an infinitesimal circulation/rotation to explain why Stokes’ theorem is true.